Excludable vs. Non-excludable Public Goods: the Price Effect on Consumption[1]

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Excludable vs. Non-excludable Public Goods: the price effect on consumption: *In this paper we provide an application of the optimization model in the process of provision and consumption of excludable and non-excludable public goods. We further analyze the effect of charging a price on the consumption of excludable public goods and the consumers' preferences. Our model focuses on the price effect of the production and provision of both types of goods. We claim that optimality in consumption is achieved only when there is equilibrium in the supply of such goods and that it is the excludable rather than the non-excludable public goods that provide for the existence of such equilibrium.*

Key words: *optimization model, duality, (non-) excludable public goods, pricing effect*

INTRODUCTION

The constraints on the public budget require the use of alternative sources of funding. Therefore, the problem of excludable public goods has been of keen interest to a number of studies found in [2], [3], [4], [5], [7], and [8], which comment on the choice between public and private provision of excludable and non-excludable public goods. On the first place, it should be noted that excludable public goods are by definition such that it is possible to charge a price or the so called “admissibility fee” for their use. Examples of excludable public goods may be various, starting from information like weather forecasts, television broadcasts, internet services and going on with non-congested roads, public beaches and parks, museums, etc. Publicly provided excludable public goods are in most cases financed by tax revenues and partly by prices whereas privately provided excludable public goods are entirely financed by fees charged for their usage.

There is a vast literature on the public and/or private provision of excludable and non-excludable public goods. Yet, the provision of excludable public goods in comparison to non-excludable public goods in terms of their nature, interrelation and typology has not been much studied. Therefore, we will confine our analysis to the study of these goods and propose an explanation on the effect of pricing and the resulting changes in consumption.

In this paper we will provide an application of the dual problem, as proposed by [6], in the analysis of public goods for which a price is charged in comparison to such goods provided free of charge and we will comment on the price effect of supplying excludable public goods. By applying the dual problem, we will analyse the optimizing behavior of the consumers with regard to excludable and non-excludable public goods and the impact of pricing on the consumption choice.

THE MODEL

1. Optimization model of excludable public goods

The starting point in our model will be the notion that excludable public goods are characterized by recurring consumption and a price is charged for each use of these goods. In this sense, one could argue that excludable public goods are similar to private goods. However, it is the nature of consumption which still differentiates between the two types of goods, viz. the excludable public good is accessed by more than one consumer while the private good is for individual use only, and the choice of the consumer is determined primarily by three factors – price, income, and leisure.

Let us firstly assume that we have a utility function $u(g^i) = u(g_1^i, g_2^i, ..., g_n^i)$, defined in a compact set $g^i \in g \subset R^n$, which is *continuous*, *monotonic*, *twice differentiable*,

[1] The paper is partially supported by the Sofia University “St. Kl. Ohridski” research project 6/2008.

quasiconcave and *homogeneous of degree 1* and characterized by *local non-satiation* which shows the level of utility achieved through the consumption of a certain private good and *n*-excludable public goods.

To formulate the utility maximization problem, we intend to introduce the value function $V^i(p,B^i)$ in the following form:

$$V^i(p,B^i) = \max_{g^i \geq 0} u^i(g^i) \qquad (1)$$
$$\text{s.t. } \langle p, g^i \rangle = w^i H^i - T^i = B^i$$

where $g^i = (g_1^i, g_2^i, ..., g_n^i)$ is a vector, consisting of the arguments of the objective function and it represents the amount of the consumed private and *n*-excludable public goods, $p = (p_1, p_2, ..., p_n)$ is a vector of prices which are positive numbers and influence the constraint. The budget constraint $B^i \geq 0$ is the consumers' income after taxation (this is the income of a certain society of consumers which we shall refer to as type *i* consumers), w^i is the wage rate, H^i is hours worked and T^i is the amount of income tax imposed.

This problem is usually solved using the Lagrangian, which for our problem (1) takes the form $L(g^i, \lambda) = u(g^i) + \lambda(B^i - \langle p, g^i \rangle)$. The solution to this problem is the vector $g^{i*}(p,B^i)$.

As the value function $V^i(p,B^i)$ is monotonically increasing with regard to B^i, then we can formulate the dual problem, i.e. for each utility level curve $u^i(g^i) = y^i$ we get the minimum value of $\langle p, g^i \rangle$, necessary for obtaining a certain level of y^i with a given price *p*. Therefore, we formulate the value function $F^i(p,y^i)$, which represents this relation and we introduce the dual problem for obtaining a minimum value of $\langle p, g^i \rangle$ under the constraint $u^i(g^i) \geq y^i$, which takes the following form:

$$F^i(p,y^i) = \min_{g^i \geq 0} \langle p, g^i \rangle \qquad (2)$$
$$\text{s.t. } u^i(g^i) \geq y^i$$

The solution to this expenditure minimization problem is the vector $h^{i*}(p,y^i)$. Applying the theorem in [6] we can prove that the point of minimum coincides with the point of maximum, i.e. the solution to the two problems is one and the same vector. It follows from the common solution of problems (1) and (2) that one can derive the following Slutsky equation:

$$\frac{\partial h_k}{\partial p_j} = \frac{\partial g_k}{\partial p_j} + g_j \frac{\partial g_k}{\partial B}, \quad (h = h^i,\ g = g^i,\ B = B_i), \quad k, j = 1, \ldots, n \qquad (3)$$

It is important to note that utility from the consumption of excludable public goods derives from the amount of the public good provided as well as from the frequency of their usage. Hence, the society of consumers is clearly divided into consumers who are willing to pay for the excludable public goods provided (our type *i* consumers) and consumers who are not willing to pay and who are thus self-excluded from the use of these goods.

In their analysis [7] argue that the consumption of excludable public goods is a function of the number of visits/utilizations of the public goods, v_i and the amount of the public goods provided. This function takes the form $g^i = \sigma(v^i, G)$ and as claimed by [7] it has the following properties:

1. $\sigma(0,G) = \sigma(v^i,0) = 0$
2. $\dfrac{\partial \sigma}{\partial v^i} > 0$
3. $\dfrac{\partial \sigma}{\partial G} > 0$
4. $\dfrac{\partial^2 \sigma}{\partial v^i \partial G} > 0$

A similar analysis can be found in [1] applied to the private contributions in the supply of public goods. The authors claim that the decision on whether or not to become a contributor ("the extensive margin") is at least as important as the decision on how much to contribute ("the intensive margin").

In fact, the number of utilizations quantifies the excludable public good being consumed and provides a precise measurement of the demand for this good. Hence, it is the vector of price, or admissibility fee charged, and the willingness to pay that determine the consumption of the good and thus make it admissible for a given society of consumers and at the same time excludes it from consumption by other people outside this society. Therefore, it is these two elements which mark the line between excludable and non-excludable public goods.

2. Excludable vs. Non-excludable public goods – the price effect on consumption

Indeed, the vector of prices is a major distinguishing factor which determines the provision of excludable public goods as they are financed from payments that people are willing to make in order to boost up their benefit. In this regard, our contribution to the existing literature is the claim that public goods are in their primal form non-excludable as all member of society can have access to them free of charge while the imposition of prices transforms these goods into excludable and available only for those consumers who are inclined to pay for their usage.

Hence, starting from this first conclusion and the solutions to problems (1) and (2) and by using the results from the application of equation (3) in [6] we will propose a classification of excludable and non-excludable public goods based on the choice of pricing in the process of their provision.

As earlier mentioned, our leading assumption is that all public goods are in their primal existence non-excludable. The modeling of non-excludable public goods is done by constraining the price to zero and recognizing the fact that each consumer type consumes a fixed amount of the good provided. Thus, our first observation will reflect on the case when the price is set to zero and the good is entirely publicly provided, i.e. we shall term such goods as purely non-excludable.

The indirect utility function representing the maximum utility attained as a result of the use of non-excludable public goods (when we set $p_{PG} = 0$) takes the following form:

$$V_{PG} = \max u(g_{PG}) \quad \text{s.t.} \ \langle \tau, g_{PG} \rangle = T \tag{4}$$

The constraint $\langle \tau, g_{PG} \rangle = T$ is the amount of government expenditure used for the provision of such goods, where τ is the lump sum taxation paid by all individuals, used for the financing of non-excludable public goods. However, we should stress here that the utility function $u(g_{PG})$ measures the level of utility obtained only from those members of society who benefit from the use of these goods. The rest of the tax payers contribute to their financing without having direct benefit from them.

The dual to problem (4) is the expenditure minimization problem, represented through the indirect cost function:

$$\phi = \min\langle \tau, g_{PG} \rangle$$
$$\text{s.t. } u(g_{PG}) \geq G_{PG}$$

It follows form the theorem in [6] that there should be only one point of optimality in the consumption of non-excludable public goods, and regardless of the level of taxes paid this point of optimality will be one and the same as the amount of the public goods consumed will not change. Hence, the consumption of non-excludable public goods is in constant equilibrium so long as lump sum taxation is enough to ensure the provision of the goods to satisfy the required preference levels of the consumers. However, our claim is that aggregate utility never reaches the solution for extremum as there is no pre-selection of the goods to be supplied based on consumers' preferences. We will propose the conclusion that one could have some reservations when speaking about optimality in such equilibrium that affects all members in a given society and that in general, the social welfare function (in our case represented in (4)) does not really reflect on the optimal utility of each individual consumer and is not a sum of individual optimal points. The economic intuition that we propose in this paper is that the social welfare function only represents the possible optimality in the consumption of non-excludable public goods taking into account the collective preferences of the consumers and that the optimal solution is truly optimal only for the "active" consumers of the goods while indeed not quite so for the "passive" ones.

Therefore, referring to the application of equation (3), in a large economy model when there is consumption of private and non-excludable public goods, the public goods are in most of the cases normal or ordinary goods. Only when the price of the private good changes – increases/decreases (substitution effect) or the consumers' income increases /decreases (income effect), the non-excludable public good may turn into a substitute/a complement in the first case and into inferior/Giffen good in the second case. There would hardly be a case when a non-excludable public good can be a luxurious good.

The second case which we will discuss is the case when there is a mixed financing of a given public good. In this example, the starting price would certainly be $p_{PG} > 0$ and the contribution of the consumers will be also in a mixed form, i.e. the lump sum tax paid in a combination with the price set for the access to the good. Such public-private provision of public goods is the most common case when the public good provided is turned into an excludable public good and the level of exclusivity greatly depends on the price which is the private element in the provision process. The constraint for this maximization problem takes the following form:

$$\langle (\tau + p), G \rangle = wH + T$$

Therefore, our postulation is that whether one considers a given public good as excludable or non-excludable depends greatly on the level of price imposed for this good. Again, by applying equation (3) we observe that when we fix the vector of prices as constant we have the case of ordinary and normal excludable public goods. Otherwise, when the price varies, we differentiate between luxurious, inferior or Giffen excludable public goods and also either substitutes or complements.

The last type of excludability which we will observe is when we have only private provision of public goods. This is the case when we would rather prefer to speak of admissibility fee than of price paid for accessing this good, which we propose to define as purely excludable public good. In this case, the utility maximization problem (1) has a solution for an extremum, i.e. the aggregate utility is indeed optimal in the consumption of the good as it is entirely privately provided and accessed only by those individuals who are willing to pay for the use of the good regardless of the level of price imposed. Therefore, in this case we will define the function in (1) excludable welfare function and in this case the constraint which is a condition for the utility maximization can be rewritten as:

$$\langle p_{pr}, G \rangle = B^i \text{ , where } p_{pr} >> 0$$

It is worth noting that when there is no change in the composition of consumers, it is rarely the case when such a good may become inferior or Giffen and quite more often it is normal/ordinary or luxurious. However, the application of equation (3) shows that both the mixed and purely excludable public goods bear many of the characteristics of private goods and only the fact that they are, after all, consumed by more than one person still requires to classify them as public goods.

CONCLUSION

In this article we modeled excludable and non-excludable public goods through the solution of the dual problem. We firstly provided an overview of excludable public goods through the implications of the pair of problems – the utility maximization problem and the expenditure minimization problem - and further on discussed the impact of prices on the consumption of these goods. We claimed that public goods are in their primal nature non-excludable and that it is the price effect which attributes to them exclusivity only for a limited number of consumers. Our contribution to the available literature treating this topic was the study of the provision of excludable public goods in comparison to non-excludable public goods in terms of their nature, interrelation and we proposed classifications of both types of goods on the basis of the solution to the dual problem and the Slutsky equation and taking into account the changes in quantities and price.

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The report is reviewed.