Discrete Dynamical Systems Generated by the Composition of Finite Mealy Automata

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Abstract: Discrete dynamical systems generated by the composition of finite Mealy automata: *This paper considers the solutions of the following questions: the construction of the composition of random number of finite Mealy automata; the construction of inverse Mealy automaton and the solution of linear automaton equations; the development of suitable algorithms and.*

Key words**:** discrete dynamical systems; Mealy automaton; composition; inverse automaton; linear automaton equation.

INTRODUCTION

Let us recall some fundamental definitions, modifying some of them, and introducing certain special notations and suitable general replacements[1],[2],[3],[4].

Definition 1.

A finite automaton M is a 6-tuple $M=(Q,V_1,V_2,\delta,\lambda,q_0)$,where $Q=\{q_0,q_1,...,q_m\}$ is finite set of control states; V - starting and exit alphabet; $\delta : Q x V_1 \to Q$ - state transition function; $\lambda : Q x V_1 \to V_2$ - outset function; $q_0 \in Q$ is the initial state of the finite control (when V_1 coincides with V_2 - Mealy automaton).

Remark: The function $[x]$ is the integer part of x.

Now lets $M_1=(Q',V,V,\delta_1,\lambda_1,q_0')$, $M_2=(Q'',V,V,\delta_2,\lambda_2,q_0'')$ are Mealy automata.

Definition 2.

The composition, $M=(Q,V,V,\delta,\lambda,q_0)$ is called composition of M_1 and M_2 and denotes

$M=M_2 \circ M_1$, where $|Q'|=m_1$, $|Q''|=m_2$, $|Q|=m_1.m_2$,

$$\delta(q,x)=m_2.(\delta_1(q',x)-1)+\delta_2(q'',\lambda_1(q',x)), \quad (1)$$

$$\lambda(q,x)=\lambda_2(q'',\lambda_1(q',x)), \quad (2)$$

as $q'=\left[\dfrac{q+m_2-1}{m_2}\right]$, $q''=q-\left[\dfrac{q-1}{m_2}\right].m_2$.

An example to make it clear is done:

Example 1.

Let the finite automata M_1 and M_2 are given as follows:

$M_1=(Q',V,V,\delta_1,\lambda_1,q_0')$, where $V=\{a,b,c\}$; $Q'=\{1,2,3,4\}$, $q_0'=1$ and its functions δ_1 and $\lambda_1$are given in fig.1.

M_1	1	2	3	4
a	2,b	3,c	4,b	1,a
b	4,c	1,a	2,c	3,b
c	3,a	4,b	1,a	2,c

fig. 1

$M_2 = (Q'', V, V, \delta_2, \lambda_2, q_0'')$, where $V = \{a,b,c\}$; $Q'' = \{1,2,3,4\}$, $q_0'' = 1$ *and its functions* δ_2 *and* λ_2 *are given in fig.2.*

M_2	1	2	3	4
a	4,b	3,c	2,a	2,b
b	2,c	4,a	1,c	3,a
c	3,a	3,b	4,b	1,c

fig. 2

After some calculation (using program 2) of the functions δ and λ according to formulae (1) and (2) the automaton $M = (Q, V, V, \delta, \lambda, q_0)$ can be defined as follows:

$V = \{a,b,c\}$; $Q = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$, $q_0 = 1$ *and its functions* δ *and* λ *are given in fig.3.* (calculations are done using suitable software developed by the author executing formulae (1) and (2)).

M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
a	6,c	8,a	5,c	5,a	11,a	11,b	12,b	9,c	14,c	16,a	13,c	15,a	4,b	1,c	2,a	2,b
b	15,a	15,b	16,b	13,c	4,b	1,c	2,a	2,b	7,a	7,b	8,b	5,c	10,c	12,a	9,c	11,a
c	12,b	9,c	10,a	11,b	14,c	16,a	13,c	15,a	4,b	1,c	2,a	2,b	7,a	7,b	8,b	5,c

fig. 3

For the word $\alpha_1 = bacac$ the two consequences with the values of $\delta_1, \lambda_1, \delta_2, \lambda_2$ and δ, λ can be found as follows:

$\delta_1(1,b) = 4$; $\lambda_1(1,b) = c$;
$\delta_1(4,a) = 1$; $\lambda_1(4,a) = a$;
$\delta_1(1,c) = 3$; $\lambda_1(1,c) = a$;
$\delta_1(3,a) = 4$; $\lambda_1(3,a) = b$;
$\lambda_1(4,c) = c$;
$\Rightarrow M_1(\alpha_1) = caabc$
$\delta_2(1,c) = 3$; $\lambda_2(1,c) = a$;
$\delta_2(3,a) = 2$; $\lambda_2(3,a) = a$;
$\delta_2(2,a) = 1$; $\lambda_2(2,a) = c$;
$\delta_2(1,b) = 2$; $\lambda_2(1,b) = c$;
$\lambda_2(2,c) = b$;
$\Rightarrow M_2(M_1(\alpha_1)) = aaccb$
$\delta(1,b) = 15$; $\lambda(1,b) = a$;
$\delta(15,a) = 2$; $\lambda(15,a) = a$;
$\delta(2,c) = 9$; $\lambda(2,c) = c$;
$\delta(9,a) = 14$; $\lambda(9,a) = c$;
$\lambda(14,c) = b$;

Then $M(\alpha_1) = aaccb = M_2(M_1(\alpha_1))$

An investigation on the word $\alpha_2 = aaccbbac$ follows:

$\delta_1(1,a)=2$; $\lambda_1(1,a)=b$;
$\delta_1(2,a)=3$; $\lambda_1(2,a)=c$;
$\delta_1(3,c)=1$; $\lambda_1(3,c)=a$;
$\delta_1(1,c)=3$; $\lambda_1(1,c)=a$;
$\delta_1(3,b)=2$; $\lambda_1(3,b)=c$;
$\delta_1(2,b)=1$; $\lambda_1(2,b)=a$;
$\delta_1(1,a)=2$; $\lambda_1(1,a)=b$;
$\lambda_1(2,c)=b$;

$\Rightarrow M_1(\alpha_2)=bcaacabb$

$\delta_2(1,b)=2$; $\lambda_2(1,b)=c$;
$\delta_2(2,c)=3$; $\lambda_2(2,c)=b$;
$\delta_2(3,a)=2$; $\lambda_2(3,a)=a$;
$\delta_2(2,a)=1$; $\lambda_2(2,a)=c$;
$\delta_2(1,c)=3$; $\lambda_2(1,c)=a$;
$\delta_2(3,a)=2$; $\lambda_2(3,a)=a$;
$\delta_2(2,b)=4$; $\lambda_2(2,b)=a$;
$\lambda_2(4,b)=a$;

$\Rightarrow M_2(M_1(\alpha_2))=cbacaaaa$

$\delta(1,a)=6$; $\lambda(1,a)=c$;
$\delta(6,a)=11$; $\lambda(6,a)=b$;
$\delta(11,c)=2$; $\lambda(11,c)=a$;
$\delta(2,c)=9$; $\lambda(2,a)=c$;
$\delta(9,b)=7$; $\lambda(9,b)=a$;
$\delta(7,b)=2$; $\lambda(7,b)=a$;
$\delta(2,a)=8$; $\lambda(2,a)=a$;
$\lambda(8,c)=a$;

Then $M(\alpha_2)=cbacaaaa=M_2(M_1(\alpha_2))$

This example gives us grounds to formulate and prove the following theorem:

Theorem 1. The composition $M = M_2 \circ M_1$ of two Mealy automata generates the automaton map $\varphi = \varphi_2 \circ \varphi_1$.

Proof

Let $M=(Q,V,V,\delta,\lambda,q_0)$ be the Mealy automaton, constructed above, $\alpha \in V^*$, $\alpha = a_{i_1}a_{i_2}a_{i_3}...a_{i_s}$,

$\delta_1(q_0',a_{i_1})=q_{i_1}'$, $\delta_1(q_{i_{k-1}}',a_{i_k})=q_{i_k}'$, $\lambda_1(q_0',a_{i_1})=b_{i_1}$, $\lambda_1(q_{i_{k-1}}',a_{i_k})=b_{i_k}$,

$\delta_2(q_0'',b_{i_1})=q_{i_1}''$, $\delta_2(q_{i_{k-1}}'',b_{i_k})=q_{i_k}''$, $\lambda_2(q_0'',b_{i_1})=c_{i_1}$, $\lambda_2(q_{i_{k-1}}'',b_{i_k})=c_{i_k}$,

$k=1,...,s$.

Thus $M_1(\alpha)=b_{i_1}b_{i_2}b_{i_3}...b_{i_s}$, $M_2(M_1(\alpha))=c_{i_1}c_{i_2}c_{i_3}...c_{i_s}$.

An investigation on a_{i_1} and a_{i_k} (for any $k = 2,...,s$) follows.

$$a_{i_1}:$$
$$\delta_1(q_0', a_{i_1}) = q_{i_1}',$$
$$\lambda_1(q_0', a_{i_1}) = b_{i_1},$$
$$\delta_2(q_0'', b_{i_1}) = q_{i_1}'',$$
$$\lambda_2(q_0'', b_{i_1}) = c_{i_1},$$

which means $M_2(M_1(a_{i_1})) = c_{i_1}$.

$$\delta(q_0, a_{i_1}) = m_2.(\delta_1(q_0', a_{i_1}) - 1) + \delta_2(q_0'', \lambda_1(q_0', a_{i_1})) =$$
$$= m_2(q_{i_1}' - 1) + \delta_2(q_0'', b_{i_1}) = m_2.(q_{i_1}' - 1) + q_{i_1}''$$
$$\lambda(q_0, a_{i_1}) = \lambda_2(q_0'', \lambda_1(q_0', a_{i_1})) = \lambda_2(q_0'', b_{i_1}) = c_{i_1}$$

and so $M(a_{i_1}) = c_{i_1} = M_2(M_1(a_{i_1}))$.

The same way can be obtained the result for a_{i_k}:

$$\delta_1(q_{i_{k-1}}', a_{i_k}) = q_{i_k}',$$
$$\lambda_1(q_{i_k}', a_{i_k}) = b_{i_k},$$
$$\delta_2(q_{i_{k-1}}'', b_{i_k}) = q_{i_k}'',$$
$$\lambda_2(q_{i_{k-1}}'', b_{i_k}) = c_{i_k},$$

which means $M_2(M_1(a_{i_k})) = c_{i_k}$.

$$\delta(q_{i_{k-1}}, a_{i_k}) = m_2.(\delta_1(q_{i_{k-1}}', a_{i_k}) - 1) + \delta_2(q_{i_{k-1}}'', \lambda_1(q_{i_{k-1}}', a_{i_k})) =$$
$$= m_2(q_{i_k}' - 1) + \delta_2(q_{i_{k-1}}'', b_{i_k}) = m_2.(q_{i_k}' - 1) + q_{i_k}''$$
$$\lambda(q_{i_{k-1}}, a_{i_k}) = \lambda_2(q_{i_{k-1}}'', \lambda_1(q_{i_{k-1}}', a_{i_k})) = \lambda_2(q_{i_{k-1}}'', b_{i_k}) = c_{i_k}.$$

Therefore $M(a_{i_k}) = c_{i_k} = M_2(M_1(a_{i_k}))$ and hence $M(\alpha) = M_2(M_1(\alpha))$ so in conclusion $M = M_2 \circ M_1$.

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