# Analysis and study on the one-hop radio-based localization techniques for wireless sensor networks

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Analysis and study on the one-hop radio-based localization techniques for wireless sensor **networks**: In this paper the principles of the one-hop localization techniques in which the non-anchor node to be localized is the one-hop neighbour of a sufficient number of anchors is being discussed. There is an analysis of angle of arrival (AOA)-based, time different of arrival (TDOA)-based, distance-based and received signal strength (RSS)-profiling based localization techniques and some existing algorithms for them. The paper is then completed by the conclusion section, followed by the acknowledgment and references sections.

Key words: Wireless sensor networks, radio-hop localization, AOA, TDOA, Distance-based, RSS

#### INTRODUCTION

The rapid rate of development in the fields of telecommunication and computer sciences has led to the emerging of several new technologies and paradigms for networking. One of these new ideas was the development of tiny mobile devices with sensing capabilities and with the possibility for wireless data delivery. Since the initial introduction of these devices, they have been found as suitable for a variety of different purposes - from animal movement and population monitoring, trough warning systems and systems for detection of hazardous agents and radiation to the latest military purposes - as vehicle and troops tracking and monitoring systems [1, 3, 8, 9]. Despite the variety of sensor devices and their many purposes, there are several disadvantages of the networks they form. One of the largest advantages of the wireless sensor networks (WSNs) is the possibility to locate and track different objects. This is also a prerequisite for the efficient work of the directed and the hierarchical routing approaches, since they rely on the location of the different neighbouring sensor devices in the sensor field.

#### AOA based one-hop localization techniques

In the absence of noise and interference, bearing lines from two or more receivers will intersect to determine a unique location, i.e., the location estimate of the transmitter. In the presence of noise, more than two bearing lines will not intersect at a single point and statistical algorithms, sometimes called triangulation or fixing methods, are required in order to obtain the location estimate of the transmitter [7]. This is shown in Fig. 1.



Fig. 1 – In the presence of noise, bearing lines from three receivers will not interact at the same point

Location estimation using bearing measurements is a well researched problem. Another well-known approach is the maximum likelihood (ML) estimator [7]. The 2D localization problem using bearing measurements can be formulated as follows. Let  $x_i = [x_i, y_i]^T$  be the true coordinate vector of the transmitter to be estimated from bearing measurements  $\beta = [\beta_1, ..., \beta_N]^T$ , where N is the total number of receivers. Let  $x_i = [x_i, y_i]^T$  be

the known location of the receiver associated with the ith bearing measurement.

Denote by  $\theta(x) = [\theta_1(x), \dots, \theta_N(x)]^T$  the bearings of a transmitter located at  $x = [x,y]^T$  at the receiver locations, where  $\theta_i(x)$ ,  $1 \le i \le N$  is related to x by:

$$\tan \theta_i(x) = \frac{y - y_i}{x - x_i} \tag{1}$$

The measured bearings of the transmitter consist of the true bearings corrupted by additive noises  $\boldsymbol{\varepsilon} = [\varepsilon_1, \ldots, \varepsilon_N]^T$ , which are assumed to be zero-mean Gaussian noises with N × N covariance matrices  $S = diag\{\sigma_1^2, ..., \sigma_N^2\}$ , i.e.,

$$\beta = \theta(x_t) + \varepsilon \tag{2}$$

When the receivers are identical and much closer to each other than to the transmitter, the variances of bearing measurement errors are equal, i.e.,  $\sigma_1^2 = ... \sigma_N^2 = \sigma^2$ 

$$\hat{x}_{t} = \arg\min\frac{1}{2} \left[\theta(\hat{x}_{t}) - \beta\right]^{T} S^{-1} \left[\theta(\hat{x}_{t}) - \beta\right]$$
(3)

$$= \arg\min\frac{1}{2}\sum_{i=1}^{N}\frac{\left(\theta_{i}(\hat{x}_{i}) - \beta_{i}\right)^{2}}{\sigma_{i}^{2}}$$
(4)

The nonlinear minimization problem in Eq. (3) can be solved by a Newton–Gauss iteration [7]

$$\hat{x}_{t,k+1} = \hat{x}_{t,k} + (\theta_x(\hat{x}_{t,k})^T S^{-1} \theta_x(\hat{x}_{t,k}))^{-1} \theta_x(\hat{x}_{t,k})^T S^{-1} [\beta - \theta_x(\hat{x}_{t,k})]$$
(5)

where  $\theta_x(\hat{x}_{t,k})$  denotes the partial derivative of  $\theta$  with respect to x evaluated at  $\hat{x}_{t,k}$ . The use of Eq. (5) requires an initial estimate close enough to the true minimum of the cost function. Such an initial estimate may be obtained from prior information, or using a suboptimal procedure [7].

#### **TDOA-based one-hop localization techniques**

Given the TDOA measurement  $\Delta t_{ij}$  and the coordinates of receivers i and j, eq.  $\Delta t_{ij} \stackrel{\Delta}{=} t_i - t_j = \frac{1}{c} (||r_i - r_i|| - ||r_j - r_i||), \quad i \neq j$  (where  $t_i$  and  $t_j$  are the time when a signal is received at receivers i and j, respectively, c is the propagation speed of the signal, and  $||\cdot||$  denotes the Euclidean norm) defines one branch of a hyperbola whose foci are at the locations of receivers i and j and on which the transmitter  $r_t$  must lie. In  $\mathbb{R}^2$ , measurements from a minimum of three receivers are required to uniquely determine the location of the transmitter. This is illustrated in Fig. 2. In a system of N receivers, there are N-1 linearly independent TDOA equations, which can be written compactly as [7]:

$$\begin{bmatrix} \|r_{1} - r_{r}\| - \|r_{N} - r_{r}\| - c\Delta t_{1N} \\ \dots \\ \|r_{N-1} - r_{r}\| - \|r_{N} - r_{r}\| - c\Delta t_{N-1;N} \end{bmatrix} = 0$$
(6)

In practice,  $\Delta t_{ij}$  is not available, instead there is the noisy TDOA measurement  $\Delta \tilde{t}_{ij}$  given by:

$$\Delta \tilde{t}_{ij} = \Delta t_{ij} + n_{ij} \tag{7}$$

where  $n_{ij}$  denotes an additive noise, which is usually assumed to be an independent zero-mean Gaussian distributed random variable. Eq. (6) is a nonlinear equation that is difficult to solve, especially when the receivers are arranged in an arbitrary fashion. Moreover, in the presence of noise, Eq. (6) may not have a solution.

A noisy version of Eq. (6) can be written as:



Fig. 2 – Intersecting hyperbolas from three receivers

Denote by  $\Delta \tilde{t}$  the TDOA measurement vector  $\left[\Delta \tilde{t}_{1N},...,\Delta \tilde{t}_{N-1N}\right]^T$ . Denote by  $f(\mathbf{r}_t)$  the vector  $\left[\frac{1}{c}(\|r_1 - r_t\| - \|r_N - r_t\|),...,\frac{1}{c}(\|r_{N-1} - r_t\| - \|r_N - r_t\|)\right]^T$  and denote by S the covariance matrix of the TDOA measurement errors. The ML estimator minimizes the following quadratic function:

$$Q(\hat{r}_t) = [\Delta \tilde{t} - f(\hat{r}_t)]^T S^{-1} [\Delta \tilde{t} - f(\hat{r}_t)]$$
(9)

in which  $f(r_t)$  is a nonlinear vector function. In order to obtain a reasonably simple estimator,  $f(r_t)$  can be linearized using Taylor series around a reference point  $r_0$ 

 $f(r_t) \approx f(r_0) + f_r(r_0)(r_t - r_0)$ (10)

where  $f_r(r_0)$  is a (N-1) ×2 (in  $R^2$ ) matrix of partial derivative of f with respect to r evaluated at  $r_0$ . A recursive solution to the ML estimator can then be obtained [7]:

 $\hat{r}_{t,k+1} = \hat{r}_{t,k} + (f_r(r_{t,k})^T S^{-1} f_r(r_{t,k}))^{-1} f_r(r_{t,k})^T S^{-1} [\Delta \tilde{t} - f(r_{t,k})]$ (11)

This method relies on a good initial guess of the transmitter location. Moreover, in some situations this method can result in significant location estimation errors due to geometric dilution of precision effects, which describes a situation in which a relatively small ranging error can result in a large location estimation error because the transmitter is located on a portion of the hyperbola far away from both receivers.

#### Distance-based one-hop localization techniques

The most well-known distance-based localization technique is based on use of GPS. The GPS space segment consists of 24 satellites in the medium earth orbit at a nominal altitude of 20,200 km with an orbital inclination of 55° [5]. Each satellite carries several high accuracy atomic clocks and radiates a sequence of bits that starts at a precisely known time. The location of a GPS satellite at any particular time instant is known. A GPS receiver located on the earth derives its distance to a GPS satellite from the difference of the time a GPS signal is received at the receiver and the time the GPS signal is radiated by the GPS satellite. Ideally, distance measurements to three GPS satellites allow the GPS receiver to uniquely determine its position. In reality, four satellites, rather than three, are required because of synchronization error in the receiver's clock. The fourth distance measurement provides information from which the synchronization error of the receiver can be corrected and the receiver's clock can be synchronized to accuracy better than 100 ns.

Generally in a WSN, for a non-anchor node at unknown location  $x_t$  with noisecontaminated distance measurements  $\tilde{d}_1,...,\tilde{d}_N$  to N anchors at known locations  $x_1, \ldots, x_N$ , the location estimation problem can be formulated using a maximum likelihood approach as:

$$\hat{x}_{t} = \arg\min[d(\hat{x}_{t}) - \tilde{d}]^{T} S^{-1}[d(\hat{x}_{t}) - \tilde{d}]$$
(12)

where  $\tilde{d}$  is a N × 1 distance measurement vector,  $d(\hat{x}_i)$  is also a N × 1 vector  $[\|\hat{x}_i - x_1\|, ..., \|\hat{x}_i - x_N\|]$  and S is the covariance matrix of the distance measurement errors.

An interesting development in the area is the use of the Cayley–Menger determinant to reduce the impact of distance measurement errors on the location estimate [4]. To illustrate the concept, consider a non-anchor node  $x_t$  having distance measurements to three anchors  $x_1$ ,  $x_2$ ,  $x_3$  in  $\mathbb{R}^2$ , which is shown in Fig. 3.

The Cayley–Menger determinant of this quadrilateral is given by:

$$D(x_1, x_2, x_3, x_t) = \begin{vmatrix} 0 & d_{12}^2 & d_{13}^2 & d_{t1}^2 & 1 \\ d_{12}^2 & 0 & d_{23}^2 & d_{t2}^2 & 1 \\ d_{13}^2 & d_{23}^2 & 0 & d_{t3}^2 & 1 \\ d_{t1}^2 & d_{t2}^2 & d_{t3}^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$
(13)



Fig. 3 – A fully connected planar quadrilateral in sensor networks

$$\varepsilon^T A \varepsilon + \varepsilon^T b + c = 0 \tag{14}$$

where  $\varepsilon = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^T$ , the matrix A, vectors b and c can be expressed in the form of known inter-anchor distances  $d_{12}$ ,  $d_{13}$ ,  $d_{23}$  and measured distances  $\tilde{d}_{r1}, \tilde{d}_{r2}, \tilde{d}_{r3}$  [7]. Eq. (14) forms an additional equality constraint on the non-anchor node position. For a non-anchor node forming m quadrilaterals with neighbouring anchors, there are m independent equations like Eq. (14).

#### **RSS-profiling based localization**

Each non-anchor node unaware of its location uses the signal strength measurements it collects, stemming from the anchor nodes within its sensing region, and thus creates its own RSS finger print, which is then transmitted to the central station. Then the central station matches the presented signal strength vector to the RSS model, using probabilistic techniques or some kind of nearest neighbour-based method, which chooses the location of a sample point whose RSS vector is the closest match to that of the non-anchor node to be the estimated location of the non-anchor node [2]. In this way, an estimate of the location of the non-anchor node can be obtained. The estimate is transmitted to the non-anchor node from the central station. Obviously, a non-anchor node could also obtain the full RSS model from the central station and perform its own location estimation.

The accuracy of this technique depends on two distinct factors: the particular technique used to build the RSS model, with the resultant quality of that model, and the

technique used to fit the measured signal strength vector from a non-anchor node into the appropriate part of the model. In comparison with distance-estimation based techniques, the RSS-profiling based techniques produce relatively small location estimation errors [2]. Using 802.11 technology, with dense sampling and a good algorithm, one can expect a median localization error (i.e., distance between the estimated location and the true location) of about 3 m and a 97th percentile error of about 9 m. With relatively sparse sampling, every 6 m, or 37 m<sup>2</sup>/sample, one can still achieve a median error of 4.5m and 95th percentile error of 12 m [7].

#### Analysis of the existing algorithms

The discussed 2D localization problem using bearing measurements can be seen also in [7], where analytical expressions for the bias and the covariance matrix of the estimation errors associated with the ML approach and with the Stanfield approach were given. It was shown that the Stanfield approach provides biased estimates even for a large number of bearing measurements and the ML approach is asymptotically unbiased at a large number of measurements. However the root mean square error of Stanfield approach is not necessarily larger than that of the ML approach.

In [7] it is given an exact solution to the hyperbolic equations in Eq. (6) when the number of TDOA measurements is equal to the number of unknown transmitter coordinates. However this solution cannot make use of extra measurements. Other techniques, that can deal with the more general situation with extra measurements include the spherical interpolation method [7], which is derived from least-squares "equation-error" minimization, and the divide and conquer method.

Numerical methods, such as the gradient descent algorithm, can be exploited to search for the solution for distance-based techniques, which gives a location estimate superior to that obtained using Eq. (12) only. The essence of using the Cayley–Menger determinant to reduce the impact of distance measurement errors is: the six edges of a planar quadrilateral are not independent [7]. This equality constraint can be exploited to reduce the impact of distance measurement errors. This idea may also extend to TDOA and AOA based localization.

The major practical obstacle in the RSS-profiling based localization is the extensive amount of profiling data required. Substantial and possibly costly initial experiments are needed to establish the model. Subsequent changes in the environment (e.g., inside a building, office occupancy can change) can affect the model, and so a static model derived from a single-shot experiment may be inadequate in some applications. There has been proposed a method of online profiling, which would reduce or possibly eliminate the amount of profiling required before deployment, but at the expense of deploying a large number of additional devices (termed "sniffers") at known locations [6]. Together with a large number of stationary emitters (anchor nodes) deployed at known locations, the "sniffers" can be used to construct and update the RSS model online. In [7] it is presented a weighted version of the RSS-profiling based localization technique, which achieves a more accurate location estimate. Denote by  $\gamma$  the signal strength vector of the nonanchor node. Denote by  $\beta_i$  and  $x_i$  the signal strength vector and the location vector of the i<sup>th</sup> sample point respectively. In the weighted version of the RSS-profiling based localization algorithm, the location estimate of the non-anchor point is given by:

$$\hat{x}_{t} = \sum_{i=1}^{N} \frac{\frac{1}{\|\gamma - \beta_{i}\|^{2}}}{\sum_{i=1}^{N} \frac{1}{\|\gamma - \beta_{i}\|^{2}}} x_{i}$$
(15)

where  $\|\gamma - \beta_i\|$  denotes the Euclidean distance between the two vectors  $\gamma$  and  $\beta_i$ , and N is the total number of sample points. Experimental evaluation showed that Ni's approach

achieves a median localization error of 1 m and a maximum localization error of 2 m, which appears to be better than others.

### CONCLUSION

This paper has provided analysis and study on angle of arrival, time difference of arrival, distance-based and received signal strength one-hop radio-based localization techniques for wireless sensor networks and some of the corresponding localization algorithms. All of these approaches have their advantages and disadvantages and are suitable for specific network applications. The general conclusion is that when designing a wireless sensor network, one has to take into consideration all of the factors that will impact on the accuracy of the localization. After that the appropriate approach can be selected based on the purpose and the requirements of the network. The literature survey work showed that an optimum algorithm could not be defined yet, and thus the choice of the suitable one has to be founded on the specificities of the situations, taking into account the size of the network, as well as the deployment methods and the expected results. So the development of new communication protocols, algorithms, designs, and services are needed.

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