Equations for movement of non-uniform turbulent flow of compressible flow

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***Abstract**: In the current work is making conclusion for movement of non-uniform turbulent compressible fluid. Approach to solving the problem is described and also is given the main characteristic equation used in the software Ansysis.*
***Key words:** turbulent flow, characteristic equation*

INTRODUCTION

Contrary to prevailing opinion on the application of Reynolds equations in software Ansysis, in this work is given the complete conclusion of the equations which are implemented in the software. At [3] is considered a solution for incompressible non-uniform flow.

MATHEMATICAL MODEL OF THE FLOW

The conclusion was made based on the basic equation of fluid mechanics [1], assuming so-called effective viscosity:

$$\mu_{ef} = \mu + \mu_T \tag{1}$$

The type of equations derived from the equations of fluid mechanics in stresses and taking into account eq.1 follows:

$$\begin{aligned}&\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + 2\frac{\partial}{\partial x}\left[(\mu+\mu_T)\frac{\partial u}{\partial x}\right] + \\ &+\frac{\partial}{\partial y}\left[(\mu+\mu_T)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[(\mu+\mu_T)\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]\end{aligned} \tag{2}$$

$$\begin{aligned}&\rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} + \rho w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left[(\mu+\mu_T)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right] + \\ &+2\frac{\partial}{\partial y}\left[(\mu+\mu_T)\frac{\partial v}{\partial y}\right] + \frac{\partial}{\partial z}\left[(\mu+\mu_T)\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]\end{aligned} \tag{3}$$

$$\begin{aligned}&\rho\frac{\partial w}{\partial t} + \rho u\frac{\partial w}{\partial x} + \rho v\frac{\partial w}{\partial y} + \rho w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial x}\left[(\mu+\mu_T)\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right] + \\ &+\frac{\partial}{\partial y}\left[(\mu+\mu_T)\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right] + 2\frac{\partial}{\partial z}\left[(\mu+\mu_T)\frac{\partial w}{\partial z}\right] \pm (\rho-\rho_{ok})g\end{aligned} \tag{4}$$

The system equations 2 ÷ 4 are processed by performing the usual mathematical rewriting which leads to:

$$\begin{aligned}&\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) + \\ &+\frac{1}{3}\mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) + \mu_t\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) + \frac{1}{3}\mu_t\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\end{aligned} \tag{5}$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+$$
$$+\frac{1}{3}\mu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+\mu_t\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{1}{3}\mu_t\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \tag{6}$$

$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+$$
$$+\frac{1}{3}\mu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+\mu_t\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{1}{3}\mu_t\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\pm(\rho-\rho_{ok})g \tag{7}$$

This system of equations in their appearance reminiscent of the system of Navier-Stokes equations, but in it there is additional terms which include turbulent viscosity μ_t:

The equation of continuity is added:

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \tag{8}$$

The equation for heat transfer is:

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial x}\left[(\lambda+\lambda_t+\lambda_л)\frac{\partial T}{\partial x}\right]$$
$$+\frac{\partial}{\partial y}\left[(\lambda+\lambda_t+\lambda_л)\frac{\partial T}{\partial y}\right]+\frac{\partial}{\partial z}\left[(\lambda+\lambda_t+\lambda_л)\frac{\partial T}{\partial z}\right]+q_v \tag{9}$$

where: c_p - specific heat at $p=const.$; λ - coefficient of thermal conductivity; λ_t - coefficient of turbulent thermal conductivity; $\lambda_л$ - radiant heat transfer coefficient; q_v - intensity of the internal heat sources.

CHARACTERISTIC EQUATION

The system equations (5÷9) come down to the characteristic equation (10), which is as follows:

$$\frac{\partial}{\partial t}(\rho\Phi)+div(\rho V\Phi)=div(\Gamma grad\Phi)+S \tag{10}$$

where: Φ s the independent variable; Γ - diffusion coefficient at Φ; S - source term of the relevant Φ.

Table 1.

Φ	Γ	S
1	0	0
u	$\mu+\mu_t$	$\Gamma\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\partial p}{\partial x}+\frac{1}{3}\frac{\partial}{\partial x}\Gamma\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)$
v	$\mu+\mu_t$	$\Gamma\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\partial p}{\partial y}+\frac{1}{3}\frac{\partial}{\partial y}\Gamma\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)$
w	$\mu+\mu_t$	$\Gamma\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{\partial p}{\partial x}+\frac{1}{3}\frac{\partial}{\partial x}\Gamma\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\pm$ $\pm(\rho-\rho_2)g$
T	$\lambda+\lambda_t+\lambda_л$	q_v

In table 1 are given the value for Γ and S for each Φ. Axis z for $\Phi = w$ is include in S and lift force $\pm(\rho - \rho_2)g$

The decision of the (10) is realized through the scheme in finite differences which is making consecutively for each Φ. In Ansysis [2] has a great selection of models for determining μ_t, which dependent of the nature of the study case.

The task of non-uniform flow of compressible or incompressible fluid is widely used in engineering practice. These are processes in heat engines, ventilation equipment and etc.

APPLICATION IN MODELING OF "BIOLOGY" FLOW

An interesting trend is the application of Fluent and Analysis programs in medicine in modelling certain flows. The program is using to determined conflicting process of flowing in the cochlea. Model study of the distribution of sound waves and their effects on nerve endings in the cochlea, taking into account the deformation of the membrane of the variable pressure. Sound waves distribution is successfully resolved in [6].

CONCLUSION

Developed in these work equations for non-uniform flow of compressible fluid, broaden knowledge of working with the software Ansysis. Internal structure of the software with the characteristic equation and its components is given in the current work. This allows influencing on basic equations in order to increase their weight in the solution, if necessary.

Literature

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