Mathematical model of occurrence and spread of fire in working premises

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The complexity and variety of the observed phenomena which characterize fire dynamics precondition the necessity of using mathematical models in the process of study. Mathematical models give the opportunity of operational, less time and resource taking, appropriate and scientific prediction of how fire develops. Therefore it is useful to work out mathematical models, which depending on the purpose - alone or relevantly grouped, give the opportunity to solve actual or specific problems.

The necessity of mathematical models

In modern construction of buildings and facilities various polymer and easily igniting materials are used. Such materials are widely used for tiling, floor coverings, insulation, soundproofing, parts and structural elements, equipment and furniture.

Some of these materials release vast amounts of smoke and heat for a very short time in the process of burning. Combining such materials with flaws in the design and the exploitation of the buildings results in highly dangerous situations when fire occurs.

When solving problems of fire safety the key issue is to have a precise and reliable method to estimate the heat and mass transfer. The complexity in working out such a model lies in the variety of factors and nonlinear solutions.

To work out a model of heat and mass transfer during fire is a complicated task itself but it is not the solution to the problem.

A real fire, as non-controllable burning process, is a complex, nonstationary 3D, heat transfer physical process accompanied by changes in the chemical composition and changes of the gas environment (medium) parameters in the premises, which process is not fully studied yet.

In the focus of burning the turbulent, convective and radiative heat and mass transfer is related to chemical reactions, heat transfer between burning gases and the side walls of the room. Existing openings and ventilation during firefighting lead to substantial differences in temperature, velocity and concentration regions of burning materials in the room, which complicates heat and mass transfer with the environment.

In the applicable technical regulations the design of systems (firefighting, smoke extraction and mechanical ventilation) is based on simplified methods for calculating heat and mass transfer during fire.

The present mathematical model is based on strict methods which give scientific prognosis of the dynamic of dangerous factors during fire and as a result provides the opportunity to optimize firefighting, smoke extracting and mechanical ventilation systems by taking into consideration the actual parameters during the occurrence, spread and evolution of fire, and also the thermophysical properties of the construction materials. Least but not last the present mathematical method helps the effective and efficient evacuation of people and valuable property.

Fields of application

The present work could yield to the improvement and update of the legislation concerning construction rules in the field of fire safety.

Fundamental motion and heat equations

The mathematical model of occurrence and spread of fire in working premises could be solved only by applying numerical methods.

Calculation the heat and mass transfer during fire, with the purpose to optimize the operation of firefighting, smoke extracting and mechanical ventilation systems, is closely connected to the theory of heat and mass transfer.

Determination of the dynamics of fire hazardous factors is based on solving 3D unstable differential equations for heat conduction through a multilayer wall (composite solids). For solid surfaces in a premise the boundary conditions in these equations are defined by a 3D regional model for calculation of fire heat and mass transfer

Fire in premises develops in complex thermo-gas dynamic conditions under a simultaneous impact of a number of factors:

- Non isometric flow (unlike the temperature of solid surfaces, side walls and gas flows), compression (gas density is not a constant value), pressure gradient, radiation, chemical reactions, biphasic processes (simultaneous running of several phases), irregularities of the surface, curvature of the surrounding surface, turbulence, type of the insulators, transition from laminar to turbulent flow.

The impact of the above listed factors leads to a substantial difference in the fire heat and mass transfer modeling using the well-researched “standard” conditions of heat transfer. Therefore, the calculating methods of heat and mass transfer in a fire should take into consideration the fire thermo-gas dynamic conditions.

The main specific features of heat and mass transfer processes in a fire are as follows:

- The highest difference between the values of the pressure in the different zones of the premise does not exceed one tenth percent of the value of the average pressure in the premise in the absence of explosion with shock wave.
- The speed of the gas flow is less than the speed of sound (in the absence of combustion and shock wave)
- The extents of gas transfer are large enough, i.e. the processes of thermal diffusion and turbulent diffusion should be taken into account

In mathematical models designed to calculate heat and mass transfer during fire in the premise the following assumptions and simplification of real thermo-gas dynamic process are made:

- In the full volume of the premise, there is a local thermodynamic and chemical balance which allows the use of the balance (steady-stay) equation
- The gas medium is a mixture of ideal gasses which gives satisfactory convergence of temperature and pressure ranges typical for fire.
- Local velocities and temperatures of the gas mixture components and of the solid (liquid) particles are equal in each point of the space, i.e. the interphase interactions are neglected.
- Coagulation and fragmentation of the smoke particles are neglected
- The chemical reaction of burning is one-phase and irreversible.
- Dissociation and ionization of the medium at high temperatures is not taken in mind
- The interaction of turbulence and radiation is neglected
- The adverse reaction of fast combustion of the burning material is neglected, i.e. the velocity of burning of the material is calculated using semi empiric relations without taking into account the current parameters of the gas medium.

Thermo- and barodiffusion are neglected

The gas medium is regarded as a viscous, heat conducting, collapsing ideal gas. The impact of the hard smoke particles is taken into consideration when the characteristics of the radiation heat transfer inside the premise is determined.

The model for calculating heat and mass transfer during fire consists of a system of basic differential equations and the mass, momentum and energy conservation laws, as well as additional equations necessary for the calculation.

All components of the model are integrated through common parameters (reverse correlation). Therefore the building of the model is an iterative process.

The model for calculating heat and mass transfer during fire is based on a system of basic differential equations and the mass, momentum and energy conservation laws. These equations are derived in details in the present paper. In particular, the x - axis is directed along the length of the premise, y – axis – along the width and z – axis – along the height. The center of this three dimensional Cartesian coordinate system is at the lower left corner of the premise. The parameter dimensions are presented in the International System of Units (SI).

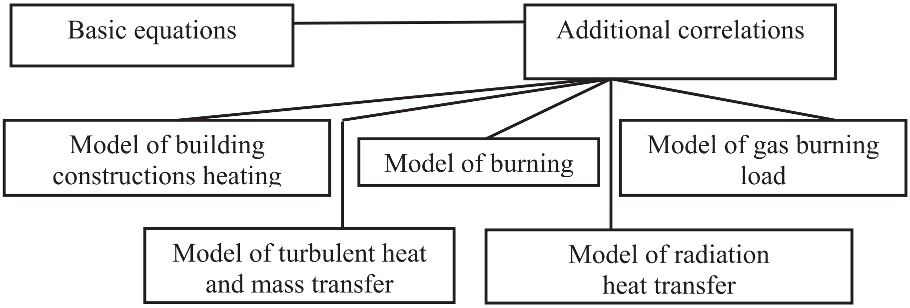

Fig.1 Structure of modeling

The Continuity equation in a mixture of gases is the mathematical representation of mass, momentum and energy conservation laws:

$$\frac{\partial\rho}{\partial\tau}+\frac{\partial}{\partial x}(\rho w_x)+\frac{\partial}{\partial y}(\rho w_y)+\frac{\partial}{\partial z}(\rho w_z)=0 \qquad (1)$$

where: **ρ** – density, [**kg/m^3**]; ***τ*** – time, [**s**]; ***x, y, z*** – coordinate axes – length, width and height of the premise in meters, [**m**]; w_x, w_y, w_z – projection of the velocity on the relevant axis, [m/s]

The vector equation of mass, momentum and energy conservation laws in a mixture of gases is presented with scalar equations along the coordinate axes $[x, y, z]$:

$$\rho\frac{\partial w_x}{\partial\tau}+\rho w_x\frac{\partial w_x}{\partial x}+\rho w_y\frac{\partial w_y}{\partial y}+\rho w_z\frac{\partial w_z}{\partial z}=-\frac{\partial p}{\partial x}+2\frac{\partial}{\partial x}\left((\mu+\mu_m)\frac{\partial w_x}{\partial x}\right)+\frac{\partial}{\partial y}\left((\mu+\mu_m)\left(\frac{\partial w_x}{\partial y}+\frac{\partial w_y}{\partial x}\right)\right)+\frac{\partial}{\partial z}\left((\mu+\mu_m)\left(\frac{\partial w_x}{\partial z}+\frac{\partial w_z}{\partial x}\right)\right)-\frac{2}{3}\frac{\partial}{\partial x}\left((\mu+\mu_m)\left(\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}+\frac{\partial w_z}{\partial z}\right)\right); \qquad (2)$$

$$\rho\frac{\partial w_y}{\partial\tau}+\rho w_x\frac{\partial w_y}{\partial x}+\rho w_y\frac{\partial w_y}{\partial y}+\rho w_z\frac{\partial w_z}{\partial z}=-\frac{\partial p}{\partial y}+2\frac{\partial}{\partial y}\left((\mu+\mu_m)\frac{\partial w_y}{\partial y}\right)+\frac{\partial}{\partial x}\left((\mu+\mu_m)\left(\frac{\partial w_x}{\partial y}+\frac{\partial w_y}{\partial x}\right)\right)+\frac{\partial}{\partial z}\left((\mu+\mu_m)\left(\frac{\partial w_y}{\partial z}+\frac{\partial w_z}{\partial y}\right)\right)-\frac{2}{3}\frac{\partial}{\partial y}\left((\mu+\mu_m)\left(\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}+\frac{\partial w_z}{\partial z}\right)\right) \qquad (3)$$

$$\rho\frac{\partial w_z}{\partial\tau}+\rho w_x\frac{\partial w_z}{\partial x}+\rho w_y\frac{\partial w_z}{\partial y}+\rho w_z\frac{\partial w_z}{\partial z}=-\frac{\partial p}{\partial y}-(\rho-\rho_0)\mathbf{g}+2\frac{\partial}{\partial z}\left((\mu+\mu_m)\frac{\partial w_z}{\partial y}\right)+\frac{\partial}{\partial x}\left((\mu+\mu_m)\left(\frac{\partial w_x}{\partial z}+\frac{\partial w_z}{\partial x}\right)\right)+\frac{\partial}{\partial y}\left((\mu+\mu_m)\left(\frac{\partial w_y}{\partial z}+\frac{\partial w_z}{\partial y}\right)\right)-\frac{2}{3}\frac{\partial}{\partial z}\left((\mu+\mu_m)\left(\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}+\frac{\partial w_z}{\partial z}\right)\right); \qquad (4)$$

where: **μ-** Coefficient of dynamic viscosity; μ_T - Coefficient of turbulent viscosity; **ρ -** Pressure; ρ_0 - Gas medium density outside the heated layer; **g -** Standard acceleration due to gravity (or standard acceleration of free fall).

Equations (1) and (4) are derived from the equations of fluid dynamics in stress by time-averaging all parameters.

The energy equation is the mathematical representation of the energy conservation and energy transfer law. The First law of thermodynamics is a version of the law of conservation of energy for thermodynamic processes and is presented as follows:

$$\rho c_p\left(\frac{\partial T}{\partial \tau}+\boldsymbol{w_x}\frac{\partial T}{\partial x}+\boldsymbol{w_y}\frac{\partial T}{\partial y}+\boldsymbol{w_z}\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial x}\left((\lambda+\lambda_m+\lambda_\pi)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left((\lambda+\lambda_m+\lambda_\pi)\frac{\partial T}{\partial y}\right)+$$

$$\frac{\partial}{\partial z}\left((\lambda+\lambda_m+\lambda_\pi)\frac{\partial T}{\partial z}\right)+q_v$$

where: **T** – temperature, [K]; $\mathbf{C_p}$ – Specific heat capacity, [J/(kg . K)]; **λ** – coefficient of thermal conductivity [W/(m . K)]; $\boldsymbol{\lambda_т}$ – coefficient of turbulent conductivity, [W/(m . K)]; $\boldsymbol{\lambda_\pi}$ – coefficient of radiation conductivity [W/(m . K)]; $\mathbf{q_v}$ – intensity of the internal heat sources [W/m^3].

$$q_v = q_{vл} + q_{vк} + q_{vг}$$

where: $q_{vл}$ - Intensity of the internal heat sources as a result of radiant heat transfer [W/m^3]; q_{vk} - Intensity of the internal heat sources as a result of convective heat transfer [W/m^3]; $q_{vг}$ - Intensity of the internal heat sources as a result of the reaction to burning [W/m^3]

Conservation law of optical density of smoke is used presented as follows:

$$\frac{\partial \boldsymbol{D_{оп}}}{\partial \tau}+w_x\frac{\partial \boldsymbol{D_{оп}}}{\partial x}+w_y\frac{\partial \boldsymbol{D_{оп}}}{\partial y}+w_z\frac{\partial \boldsymbol{D_{оп}}}{\partial z}=q_D \quad (7)$$

where: $D_{оп}$ – Optical density of smoke, Нп/ m;(Neper per meter); q_D – Intensity of the optical density of smoke from the internal source originated as a reaction to burning [W/m^3]

There are other models of burning described in the technical literature – three phase reaction, for example. Nevertheless, complicating the model of burning for given boundary values of fire-resistance of building materials does not increase the accuracy of calculations.

The conditions for unique solution of the basic system of equations and additional ratios are geometric, physical, boundary and initial conditions.

It is agreed that the geometric conditions are the coordinates of:

- the boundary surface of the surrounding walls of the premise
- the boundary surface of bulky objects in the premise;
- the border between seeable (uncovered) and hidden (which could be uncovered because of the thermal effect) gaps
- the border of the uncovered surface of the burning material or the gas leak source.

The Physical conditions are :

- gas medium thermophysical properties;
- thermophysical properties of the surrounding construction material;
- the thermophysical properties of the burning material.

The boundary conditions are defined on the inside surface of the building construction.

CONCLUSION

The mathematical model of occurrence and spread of fire in working premises could be used in design and construction companies. It could also be used by fire safety authorities to improve or optimize firefighting and smoke extracting and mechanical ventilation systems in different types of buildings - garages included, in order to ensure safe evacuation of people and valuable property.

Literature

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