Explicit dynamics of gear pair using finite element model

Svetlin Stoyanov

Abstract: *A spur gear pair model is created. The involute profile is interpolated by a spline, passing through four points. The bearings supports are modelled with discrete elements that have elastic and dissipative properties. The model is used for investigating the translational vibrations of the gear pinions. Results for the vibration acceleration, velocity, and displacement are obtained. This gives a basis for smoothing the gears work, and to analysing the vibrations of faulty gears, for example gears with cracked or broken tooth.*

Key words: *Gear Pair, Finite Element Model, Explicit Dynamics, High Frequency Vibrations, Vibration Diagnostic.*

1. Introduction

1.1. Introduction to vibration diagnostic

Machine monitoring aims to survey the machine health at critical locations, e.g. gears and bearings, and predict a future failure. A scheduled stop for maintenance can be made and the damaged element can be replaced at a certain stage of defect progress. Thus, the production runs without unexpected delays. Machine vibration monitoring uses the so called signature analysis, i.e. the characteristic vibration signature of the monitored machine element is investigated. To obtain such signatures, it is needed nonlinear models [10] and advanced methods to be used like Finite Element Method [4, 12, 1, 7], Hilbert Transform and Wavelet analysis.

In [10] is presented an investigation of the characteristic vibration signature of polyharmonic force with significant difference in the frequencies of the harmonics, when it acts on a machine element with nonlinear stiffness. For an example, this can occur in gear pairs when there is a cracked tooth in one of the gear.

This paper aims to create a finite element model of a spur gear pair without faults. The model will be used to simulate the translational vibration of gear pinions. This is the first step to investigating a gear pair with a cracked or broken tooth.

The vibration acceleration and displacement will be obtained. After that, they will be verified trough comparison with theoretical end experimental results. The theoretical results from [4] and [12] will be used. In the most experimental investigations, the accelerometer is attached on the corpus of the transmission [5, 9]. In [2], the accelerometer is attached directly to the gear. Therefore, results from [2] will be used for comparison.

1.2. Introduction to explicit dynamics

The gear trains are mechanical systems that are subjected on transient excitation. When one of the mated teeth has a crack, even more when the gear has a broken tooth, the excitation is highly dynamic due to impacts. The time of the impact evens is very short and therefore the equation system integration method has to use very small time steps.

The literature on the various methods available to solving transient problems is vast. The widely used method is so-called direct integration. The direct integration can be implicit or explicit. Implicit methods are more effectively for a relatively slow phenomenon, and explicit methods are more efficient for a very fast phenomenon, such as above mentioned impact.

Due to the highly dynamic character of the events investigated in this paper, the central difference method [8] is used. This method is explicit and it is conditionally stable. The stability limit is defined in terms of the highest frequency of the system. An approximation to the stability limit is the smallest transit time of a dilatational wave across any of the elements in the mesh.

2. Supports and loading

In this investigation, the supports are modelled with linear stiffness *k* and linear viscous friction *c*, as it is sown in Fig. 1. In addition, it is applied a linear viscous friction c_ω on gear's rotation degree. The driving gear rotates with constant angular velocity ω_1. On the driven gear is applied a constant resistance torque T_R.

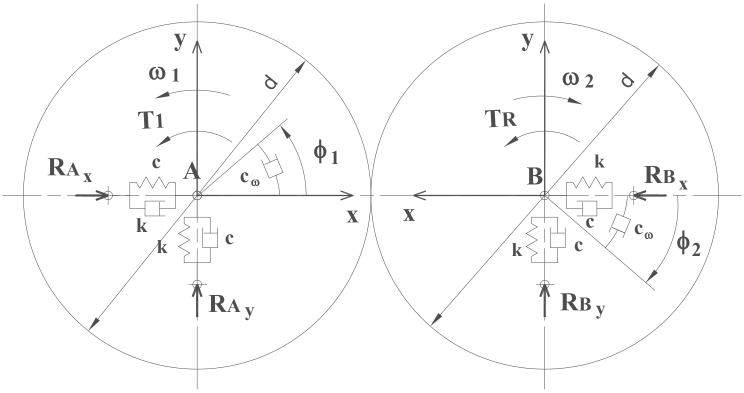

Fig. 1. Supports and loading scheme

According to [3], bearing's stiffness coefficient is usually in the range of 50 to 600 kN/mm. In this investigation is used k = 300 kN/mm. According to [6], the bearings viscous friction coefficient is chosen to be c = 1 Ns/mm. The viscous friction coefficient on the rotational degree is assumed to be c_n = 10 Nmms/rad. The angular velocity is chosen to be ω_1 = 50 rad/s, and the resistance torque – T_R = 100 Nm.

3. Involute profile creation

The involute curve generation is based on four points. The i-th point is defined as intersection point of a circle with diameter Di and a line which makes an angle

$$a_i = \sqrt{\frac{D_i^2}{D_{i-1}^2} - 1} - \arctan\left(\sqrt{\frac{D_i^2}{D_{i-1}^2} - 1}\right)$$

as it is shown on Fig. 2.

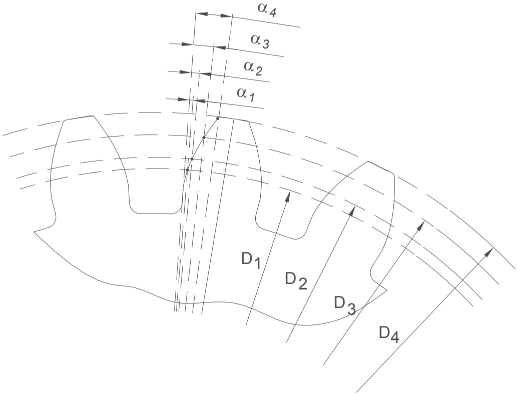

Fig. 2. Illustration of the involute profile creation method

4. Modeling of the contact

A kinematic contact formulation is used. That means that, the algorithm achieves precise compliance with the contact conditions using a predictor/corrector method [11]. At first, the increment proceeds under the assumption that contact does not occur. If at the end of the increment there is an overclosure, the acceleration is modified to obtain a corrected configuration in which the contact constraints are enforced.

The contact damping is defined to model the energy loss due to the contact presence and most of all the oil film on the interacting surfaces of gears teeth. Due to the damping presence, a resistant force occurs. The direction of this force is normal to the interacting surfaces. The value of the resistant force is proportional to the relative velocity between the surfaces. In this investigation, it is specified a unitless damping coefficient in terms of the fraction of critical damping associated with the contact stiffness.

5. Results

For the calculation process, the FE model is decomposed into four domains. Every of them contain about 3000 elements, and is processed by one processor core at 3.6 GHz. The stable time increment is $9x10^{-8}$ s, and this took about 18 hours to calculate 0.2 s.

The stable time increment is too small, and if the data has been written into file at every step, the file will becomes very large – thousands of gigabytes. Therefore, the data is written less frequently – 10^{-5} s. On the Fig. 3, the pinion vibration acceleration of the driven gear is presented. Respectively, the vibration displacement is shown on Fig. 4. The corresponding spectrograms are presented on Fig. 5. Additionally, a support reaction and stress color map are shown on Fig. 6 and Fig. 7.

6. Conclusions

After comparing with the acceleration from [4, 2] and with the displacement from [12], the results, obtained in this paper can be considered acceptable.

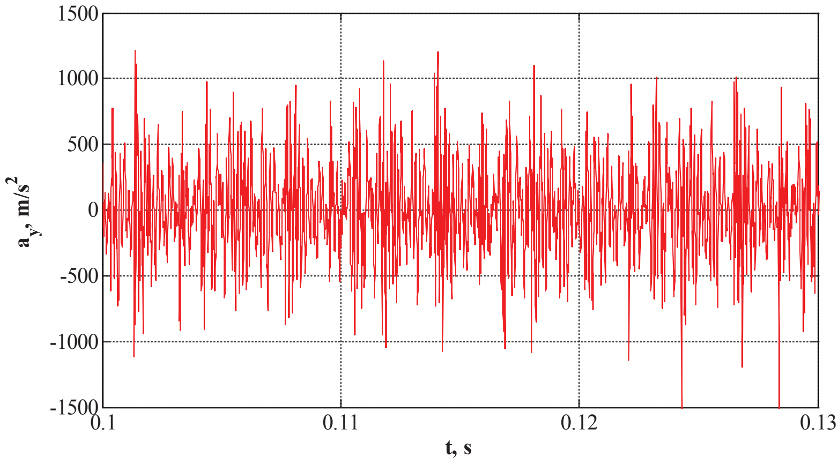

Fig.3. Time-diagram of the vibration acceleration

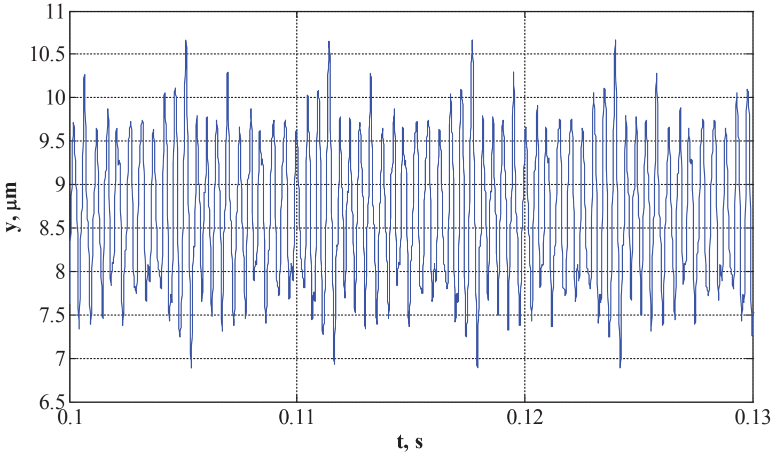

Fig. 4. Time diagram of the vibration displacement

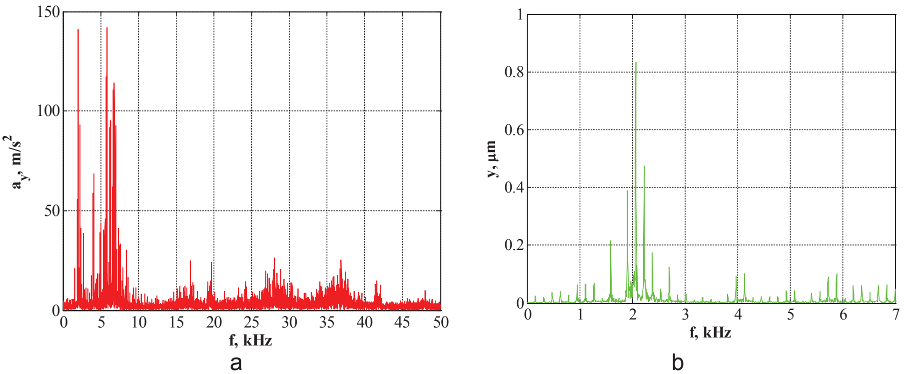

Fig. 5. Spectrograms: a – acceleration, b – displacement

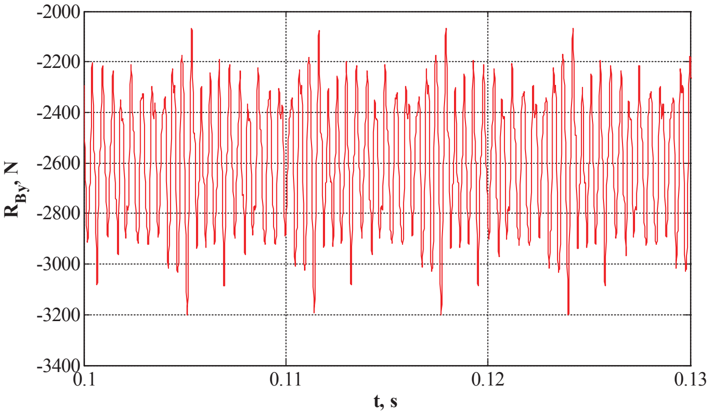

Fig. 6. Time-diagram of the driven gear support reaction force

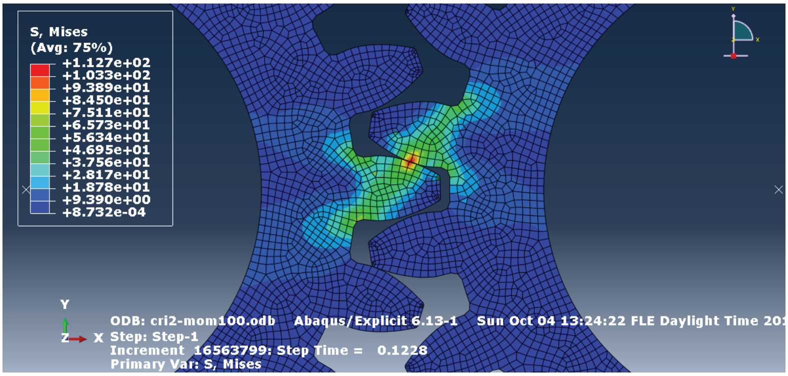

Fig. 7. Stress colour map

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ABOUT THE AUTHOR

PhD Eng. Svetlin Petrov Stoyanov, Department of Technical Mechanics, University of Ruse, E-mail: sstoyanov@uni-ruse.bg

This paper has been reviewed.