Model of Finite Element of a Plane Circumference Clamped - Joint

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Abstract: *It is investigated the stiffness matrix of the arc finite element of a plane circumference with different supported at the ends. As a basic independent parameters are assumed the displacements on the tangent and on the normal to the axis of the element and the rotations in their joints. For the background is used curvelinear arc finite element with polynomial approximation and six degree of freedoms. The investigation is made through reduction of a forces in the source element. The elements are applied for solving of systems with axis arc of circumference.*
Key words: *arc finite element, plane circumference, stiffness matrix*

STATE OF THE PROBLEM

The stiffness matrix of the curvelinear finite element, defined into circumference with used of a polynomial approximation with finite element method is investigated. The element is with six degree of freedoms. As a basic independent parameters are assumed the displacements on the tangent and on the normal to the axis of the element and the rotations in their joints.

PURPOSE OF THE STUDY

A purpose of the present investigation is through reduction of a forces in the stiffness matrix of the bilateral clamped element to receive an expressions for the components of the stiffness matrix of the curvelinear element outlined from circumference, clamped in the once end and with joint in the other end. The element to apply for solving of systems with axis – arc of circumference.

INVESTIGATION OF THE STIFNESS MATRIX OF THE ELEMENT

The basic independent parameters of the source element are the displacements on the tangent and on the normal to the axis of the element and the rotations in their joints. The element is with six degree of freedoms. It is used curvelinear coordinate system (s,φ) – tangent s, central angle φ, concerning to unspecified point of a curvelinear element.

The vector of independent parameters and of corresponding them joint's forces of an element are the kind

$$\{Z\}^T = \{u_i \quad v_i \quad \varphi_i \quad u_j \quad v_j \quad \varphi_j\}^T \qquad \{R\}^T = \{R_{i_u} \quad R_{i_v} \quad M_i \quad R_{j_u} \quad R_{j_v} \quad M_j\}^T. \tag{1}$$

For investigation of the stiffness matrix is uses a reduction of forces in the source element.

Investigation of the stiffness matrix of the curvelinear element clamped – joint

The geometry of the source element and the independent joint's displacements with their positive directions are shown in fig. 1.

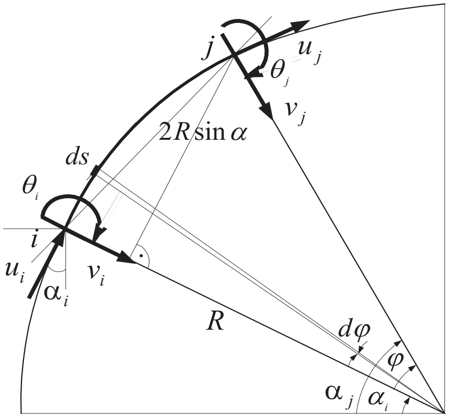

Fig. 1

It is used the following basic significations

R - radius of the element; λ - length of the element;

φ - a central angle of the element (it is mesures from the begining joint of the element);

A, I – cross section and moment of inertia of the section of the element.

In the local stiffness matrix $[k]$ of the bilateral clamped element with six degree of freedoms, after putting a moment $M_j = 0$, is receives an expression for the rotation φ_j.

$$\varphi_j = \frac{35}{4}\frac{R\left(A\lambda^2+12I\right)}{\left(A\lambda^4+420IR^2\right)}u_i + \frac{1}{4}\frac{\left(13A\lambda^4-2520IR^2\right)}{\lambda\left(A\lambda^4+420IR^2\right)}v_i + \frac{3}{4}\frac{\left(A\lambda^4-280IR^2\right)}{\left(A\lambda^4+420IR^2\right)}\varphi_i - \frac{35}{4}\frac{R\left(A\lambda^2+12I\right)}{\left(A\lambda^4+420IR^2\right)}u_j + \frac{1}{2}\frac{\left(11A\lambda^4+1260IR^2\right)}{\lambda\left(A\lambda^4+420IR^2\right)}v_j \quad (2)$$

In the vector of the joint displacements the rotation φ_j is presents through the rested joint displacements.

$$\{z\} = \begin{bmatrix} u_i \\ v_i \\ \varphi_i \\ u_j \\ v_j \\ \frac{35}{4}\frac{R\left(A\lambda^2+12I\right)}{\left(A\lambda^4+420IR^2\right)}u_i + \frac{1}{4}\frac{\left(13A\lambda^4-2520IR^2\right)}{\lambda\left(A\lambda^4+420IR^2\right)}v_i + \frac{3}{4}\frac{\left(A\lambda^4-280IR^2\right)}{\left(A\lambda^4+420IR^2\right)}\varphi_i - \frac{35}{4}\frac{R\left(A\lambda^2+12I\right)}{\left(A\lambda^4+420IR^2\right)}u_j + \frac{1}{2}\frac{\left(11A\lambda^4+1260IR^2\right)}{\lambda\left(A\lambda^4+420IR^2\right)}v_j \end{bmatrix}. \quad (3)$$

After a matrix multiplication $[r]=[k][z]$ are receive in an analytical kind the expressions for the reduced stiffness matrix of the element clamped – joint.

After transformations they accept a kind

$$k_{1,1} = \frac{\left(AR^2+I\right)}{R^2\lambda} - \frac{35}{48}\frac{1}{\lambda}\frac{\left(A\lambda^2+12I\right)^2}{\left(A\lambda^4+420IR^2\right)}; \qquad k_{1,2} = \frac{A}{2R} - \frac{1}{48}\frac{\left(13A\lambda^4-2520IR^2\right)}{R\lambda^2}\frac{\left(A\lambda^2+12I\right)}{\left(A\lambda^4+420IR^2\right)};$$

$$k_{1,3} = \frac{1}{48}\left(A\lambda^2+12I\right)\frac{\left(A\lambda^4+2520IR^2\right)}{R\lambda\left(A\lambda^4+420IR^2\right)}; \qquad k_{1,4} = -\frac{\left(AR^2+I\right)}{R^2\lambda} + \frac{35}{48}\frac{1}{\lambda}\frac{\left(A\lambda^2+12I\right)^2}{\left(A\lambda^4+420IR^2\right)};$$

$$k_{1,5} = \frac{A}{2R} - \frac{1}{24}\frac{\left(11A\lambda^4+1260IR^2\right)}{R\lambda^2}\frac{\left(A\lambda^2+12I\right)}{\left(A\lambda^4+420IR^2\right)}; \; k_{1,6} = 0;$$

$$k_{2,2}=\frac{1}{35}\frac{\left(13A\lambda^4+420IR^2\right)}{R^2\lambda^3}-\frac{1}{1680}\frac{1}{R^2\lambda^3}\frac{\left(13A\lambda^4-2520IR^2\right)^2}{\left(A\lambda^4+420IR^2\right)};$$
$$k_{2,3}=\frac{1}{210}\frac{\left(11A\lambda^4+1260IR^2\right)}{R^2\lambda^2}-\frac{1}{560}\frac{\left(A\lambda^4-280IR^2\right)\left(13A\lambda^4-2520IR^2\right)}{R^2\lambda^2\left(A\lambda^4+420IR^2\right)};$$
$$k_{2,4}=-\frac{A}{2R}+\frac{1}{48}\frac{\left(A\lambda^2+12I\right)\left(13A\lambda^4-2520IR^2\right)}{R\lambda^2\left(A\lambda^4+420IR^2\right)};$$
$$k_{2,5}=\frac{3}{70}\frac{\left(3A\lambda^4-280IR^2\right)}{R^2\lambda^3}-\frac{1}{840}\frac{\left(11A\lambda^4+1260IR^2\right)\left(13A\lambda^4-2520IR^2\right)}{R^2\lambda^3\left(A\lambda^4+420IR^2\right)};\quad k_{2,6}=0 \tag{4}$$
$$k_{3,3}=\frac{1}{105}\frac{\left(A\lambda^4+420IR^2\right)}{R^2\lambda}-\frac{3}{560}\frac{1}{R^2\lambda}\frac{\left(A\lambda^4-280IR^2\right)^2}{\left(A\lambda^4+420IR^2\right)};\qquad k_{3,4}=-\frac{1}{48}\left(A\lambda^2+12I\right)\frac{\left(A\lambda^4+2520IR^2\right)}{R\lambda\left(A\lambda^4+420IR^2\right)};$$
$$k_{3,5}=\frac{1}{420}\frac{\left(13A\lambda^4-2520IR^2\right)}{R^2\lambda^2}-\frac{1}{280}\frac{\left(11A\lambda^4+1260IR^2\right)\left(A\lambda^4-280IR^2\right)}{R^2\lambda^2\left(A\lambda^4+420IR^2\right)};\quad k_{3,6}=0$$
$$k_{4,4}=\frac{\left(AR^2+I\right)}{R^2\lambda}-\frac{35}{48}\frac{1}{\lambda}\frac{\left(A\lambda^2+12I\right)^2}{\left(A\lambda^4+420IR^2\right)};\qquad k_{4,5}=-\frac{A}{2R}+\frac{1}{24}\frac{\left(11A\lambda^4+1260IR^2\right)\left(A\lambda^2+12I\right)}{R\lambda^2\left(A\lambda^4+420IR^2\right)};\quad k_{4,6}=0;$$
$$k_{5,5}=\frac{1}{35}\frac{\left(13A\lambda^4+420IR^2\right)}{R^2\lambda^3}-\frac{1}{420}\frac{1}{R^2\lambda^3}\frac{\left(11A\lambda^4+1260IR^2\right)^2}{\left(A\lambda^4+420IR^2\right)};\qquad k_{5,6}=0;\qquad k_{6,6}=0;$$

The writeble expressions are refer only to the upper triangle part of the stiffness matrix ($k_{ij}=k_{ji}$), (a basic diagonal and the part upper him).

The solution is makes into general scheme of the FEM.

NUMERICAL REALIZATION

With investigated expressions through combinated of the element clamped-joint with element clamped-clamped, is made a solution of the following systems

Bilateral arc – circumference *with the following characteristics* [1] (fig. 2)

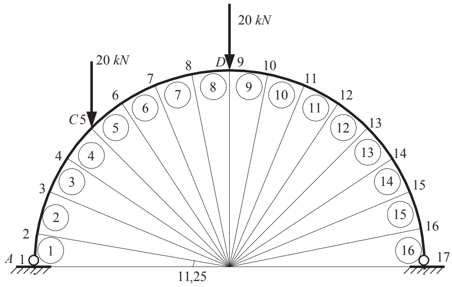

Fig. 2

Radius - $R=10\ m$. Cros section – rectangle, with dimentions $b=0{,}8\ m$, $h=1{,}6\ m$.

Modul of elasticity and coefficient of Poisson's of matherial – are respectively - $E=2{,}5.10^7\ kN/m^2$, $\nu=0{,}167$.

The solution with arc finite element is made in used of 8, 16 and 32 elements. The results obtained are shoun in table 1.

Table 1a.

Joint №	Displacements								
	Number of elements								
	8			16			32		
	$u_i(x)$	$v_i(x)$	$\varphi_i(x)$	$u_i(x)$	$v_i(x)$	$\varphi_i(x)$	$u_i(x)$	$v_i(x)$	$\varphi_i(x)$
1	0,000	0,000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000
2							7,9470e-7	8,8684e-7	1,3354e-6
3				-1,938e-6	1,586e-6	6,607e-7	2,5847e-6	2,0371e-6	2,9796e-6
4							6,0548e-6	3,8261e-6	5,1782e-6
5	-6,670e-6	3,295e-6	2,641e-6	2,388e-6	4,868e-6	5,250e-6	1,1443e-5	6,7077e-6	7,5226e-6

6							1,8554e-5	1,1058e-5	9,6012e-6
7				1,352e-5	1,292e-5	9,041e-6	2,6812e-5	1,7028e-5	1,1006e-5
8							3,5351e-5	2,4416e-5	1,1339e-5
9	6,712e-6	1,679e-5	6,566e-6	2,648e-5	2,571e-5	8,973e-6	4,3133e-5	3,2559e-5	1,0217e-5
10							4,9615e-5	4,0237e-5	8,2674e-6
11				3,585e-5	3,820e-5	6,216e-6	5,4525e-5	4,7456e-5	6,3457e-6
12							5,8037e-5	5,4006e-5	4,2976e-6
13	1,563e-5	3,233e-5	3,759e-6	4,092e-5	4,912e-5	3,195e-6	6,0277e-5	5,9556e-5	1,9698e-6
14							6,1379e-5	6,3634e-5	-7,8667e-7
15				4,236e-5	5,590e-5	-1,487e-6	6,1546e-5	6,5613e-5	-4,1146e-6
16							6,1115e-5	6,4730e-5	-8,1476e-6
17	1,636e-5	3,872e-5	-4,203e-6	4,166e-5	5,324e-5	-9,203e-6	6,0617e-5	6,0105e-5	-1,3008e-5
18							6,0771e-5	5,1249e-5	-1,7392e-5
19				4,278e-5	3,701e-5	-1,561e-5	6,2188e-5	3,9457e-5	-2,0002e-5
20							6,5085e-5	2,6368e-5	-2,0980e-5
21	2,020e-5	1,550e-5	-9,945e-6	4,849e-5	1,561e-5	-1,639e-5	6,9284e-5	1,3425e-5	-2,0489e-5
22							7,4296e-5	1,7999e-6	-1,8716e-5
23				5,690e-5	-2,121e-6	-1,294e-5	7,9413e-5	-7,6428e-6	-1,5864e-5
24							8,3801e-5	-1,4376e-5	-1,2152e-5
25	2,973e-5	-2,759e-6	-4,557e-6	6,328e-5	-1,158e-5	-6,734e-6	8,6597e-5	-1,8207e-5	-7,8103e-6
26							8,7000e-5	-1,9260e-5	-3,0792e-6
27				6,253e-5	-1,240e-5	7,327e-7	8,4352e-5	-1,7936e-5	1,7966e-6
28							7,8213e-5	-1,4853e-5	6,5704e-6
29	2,545e-5	-2,953e-6	3,620e-6	5,122e-5	-7,612e-6	7,944e-6	6,8410e-5	-1,0773e-5	1,0997e-5
30							5,5077e-5	-6,5213e-6	1,4835e-5
31				2,914e-5	-1,987e-6	1,339e-5	3,8669e-5	-2,8948e-6	1,7852e-5
32							1,9957e-5	-5,6744e-7	1,9828e-5
33	0,000	0,000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000

Table 1b.

Joint №	**Forces**								
	Number of elements								
	8			**16**			**32**		
	$N_i(x)$	$Q_i(x)$	$M_i(x)$	$N_i(x)$	$Q_i(x)$	$M_i(x)$	$N_i(x)$	$Q_i(x)$	$M_i(x)$
1	29,862	10,005	0,0000	27,017	9,444	0,0000	27,0284	9,5094	0,0000
2							-27,0289 27,0289	-6,8142 6,8142	8,0349 -8,0349
3				-26,916 26,916	-3,994 3,994	13,017 -13,017	-27,0289 27,0289	-4,0531 4,0531	13,4219 -13,4219
4							-27,0289 27,0289	-1,2523 1,2523	16,1281 -16,1281
5	-26,44 26,44	2,131 -2,131	13,246 -13,246	-26,916 26,916	1,593 -1,593	15,953 -15,953	-27,0289 27,0289	1,5619 -1,5619	16,1363 -16,1363
6							-27,0289 27,0289	4,3629 -4,3629	13,445 -13,445
7				-26,916 26,916	7,139 -7,139	8,863 -8,863	-27,0289 27,0289	7,1243 -7,1243	8,0681 -8,0681
8							-27,0289 27,0289	9,8198 -9,8198	0,0355 -0,0355
9	-26,44 12,298	12,772 1,37	-6,883 6,883	-26,916 12,774	12,445 1,697	-8,447 8,447	-27,0289 12,8868	12,4235 1,7186	-10,6073 10,6073
10							-12,8868 12,8868	-0,55 0,55	-9,261 9,261
11				-12,774 12,774	0,678 -0,678	-5,929 5,929	-12,8868 12,8868	0,6259 -0,6259	-9,114 9,114
12							-12,8868 12,8868	1,7973 -1,7973	-10,1688 10,1688
13	-12,298 12,298	3,33 -3,33	-2,907 2,907	-12,774 12,774	3,051 -3,051	-8,625 8,625	-12,8868 12,8868	2,9526 -2,9526	-12,4205 12,4205

14							-12,8868 12,8868	4,0798 -4,0798	-15,8564 15,8564
15				-12,774 12,774	5,319 -5,319	-16,623 16,623	-12,8868 12,8868	5,1676 -5,1676	-20,4558 20,4558
16							-12,8868 12,8868	6,2046 -6,2046	-26,1908 26,1908
17	-12,298 12,298	7,713 12,287	-23,104 23,104	-12,774 12,774	7,375 12,625	-30,085 30,085	-12,8868 12,8868	7,1799 12,8201	-33,0253 33,0253
18							-12,8868 12,8868	-11,8208 11,8208	-21,3175 21,3175
19				-12,774 12,774	-10,503 10,503	-9,998 9,998	-12,8868 12,8868	-10,7121 10,7121	-10,7323 10,7323
20							-12,8868 12,8868	-9,5052 9,5052	-1,3285 1,3285
21	-12,298 12,298	-7,793 7,793	2,751 -2,751	-12,774 12,774	-8,037 8,037	4,891 -4,891	-12,8868 12,8868	-8,2119 8,2119	6,8422 -6,8422
22							-12,8868 12,8868	-6,8447 6,8447	13,736 -13,736
23				-12,774 12,774	-5,325 5,325	14,994 -14,994	-12,8868 12,8868	-5,4161 5,4161	19,3163 -19,3163
24							-12,8868 12,8868	-3,9394 3,9394	23,5537 -23,5537
25	-12,298 12,298	-2,633 2,633	10,977 -10,977	-12,774 12,774	-2,459 2,459	20,619 -20,619	-12,8868 12,8868	-2,4277 2,4277	26,4262 -26,4262
26							-12,8868 12,8868	-0,8945 0,8945	27,9191 -27,9191
27				-12,774 12,774	0,476 -0,476	21,952 -21,952	-12,8868 12,8868	0,6464 -0,6464	28,0247 -28,0247
28							-12,8868 12,8868	2,1816 -2,1816	26,7425 -26,7425
29	-12,298 12,298	2,672 -2,672	10,226 -10,226	-12,774 12,774	3,396 -3,396	19,042 -19,042	-12,8868 12,8868	3,6975 -3,6975	24,0791 -24,0791
30							-12,8868 12,8868	5,1804 -5,1804	20,0486 -20,0486
31				-12,774 12,774	6,216 -6,216	11,799 -11,799	-12,8868 12,8868	6,6171 -6,6171	14,6718 -14,6718
32							-12,8868 12,8868	7,9946 -7,9946	7,977 -7,977
33	-12,298	7,763	0,0000	-12,774	8,848	0,0000	-12,8868	9,2999	0,0000

The same system is solved with bilateral clamped curvelinear element trought introduced joints in the supports. The results of the conducted two solutions are identical.

Arc – circumference, clamped in his ends and with joint in the middle, have a geometry and loading shown at fig.3.

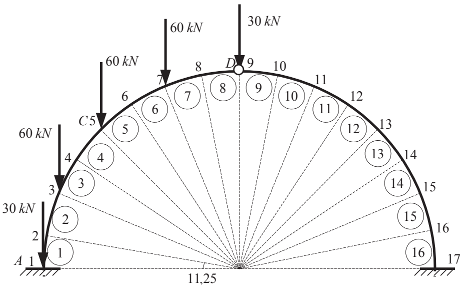

Fig. 3

Radius - $R = 10\ \ m$.

The cross section of the arc – rectangle, is with dimentions $b = 0{,}8\,m$, $h = 1{,}6\,m$. Modul of elasticity and coefficient of Poisson's of matherial – are respectively - $E = 2{,}5.10^7\ kN/m^2$, $\nu = 0{,}167$.

The solution is made in used of 8, 16 и 32 elements. The results obtained of the solution with are finite element are shown in table 2a.

Table 2a.

Joint №	Displacements								
	Number of elements								
	8			**16**			**32**		
	$u_i(x)$	$v_i(x)$	$\varphi_i(x)$	$u_i(x)$	$v_i(x)$	$\varphi_i(x)$	$u_i(x)$	$v_i(x)$	$\varphi_i(x)$
1	0,000	0,000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000
2							-1,1215e-6	5,3051e-6	-1,4491e-7
3				1,833e-6	1,097e-5	6,967e-6	-7,2387e-7	1,0931e-5	4,9207e-6
4							4,9012e-6	1,8100e-5	1,2694e-5
5	1,861e-4	5,871e-5	8,285e-5	2,324e-5	2,915e-5	2,215e-5	1,6825e-5	2,8309e-5	2,0628e-5
6							3,4837e-5	4,1123e-5	2,7752e-5
7				6,534e-5	6,033e-5	3,360e-5	5,7342e-5	5,9117e-5	3,3959e-5
8							8,2292e-5	8,2345e-5	3,7486e-5
9	4,573e-4	2,405e-4	5,938e-5	1,131e-4	1,088e-4	3,416e-5	1,0655e-4	1,0929e-4	3,6614e-5
10							1,2801e-4	1,3639e-4	3,2648e-5
11				1,484e-4	1,581e-4	2,356e-5	1,4507e-4	1,6322e-4	2,7592e-5
12							1,5739e-4	1,8810e-4	2,0436e-5
13	5,659e-4	3,808e-4	-2,436e-5	1,645e-4	1,960e-4	5,320e-6	1,6457e-4	2,0827e-4	1,0199e-5
14							1,6757e-4	2,2090e-4	-1,5623e-7
15				1,653e-4	2,067e-4	-1,242e-5	1,6772e-4	2,2779e-4	-7,3132e-6
16							1,6662e-4	2,3130e-4	-1,1938e-5
17	5,697e-4	3,390e-4	0,000	1,620e-4	2,024e-4	0,000	1,6506e-4	2,3332e-4	0,0000
18							1,6698e-4	1,5887e-4	-9,2579e-5
19				1,711e-4	7,513e-5	-7,849e-5	1,7543e-4	8,9279e-5	-8,6901e-5
20							1,8861e-4	2,8608e-5	-7,7049e-5
21	6,536e-4	-7,977e-5	-1,466e-4	1,955e-4	-2,151e-5	-5,766e-5	2,0394e-4	-2,0243e-5	-6,3929e-5
22							2,1839e-4	-5,5655e-5	-4,8491e-5
23				2,166e-4	-7,147e-5	-2,854e-5	2,2891e-4	-7,7289e-5	-3,1715e-5
24							2,3276e-4	-8,6005e-5	-1,4599e-5
25	6,698e-4	-2,034e-4	2,128e-6	2,142e-4	-7,640e-5	1,880e-6	2,2785e-4	-8,3692e-5	1,8580e-6
26							2,1307e-4	-7,3034e-5	1,6669e-5
27				1,770e-4	-5,200e-5	2,667e-5	1,8843e-4	-5,7199e-5	2,8871e-5
28							1,5527e-4	-3,9498e-5	3,7542e-5
29	3,524e-4	-5,240e-5	1,212e-4	1,098e-4	-2,068e-5	3,899e-5	1,1629e-4	-2,2996e-5	4,1811e-5
30							7,5547e-5	-1,0123e-5	4,0874e-5
31				3,646e-5	-1,729e-6	3,221e-5	3,8290e-5	-2,2777e-6	3,4002e-5
32							1,0785e-5	5,2866e-7	2,0557e-5
33	0,000	0,000	0,000	0,000	0,000	0,000	0,0000	0,0000	0,0000

Table 2b

Joint №	Forces								
	Number of elements								
	8			**16**			**32**		
	$N_i(x)$	$Q_i(x)$	$M_i(x)$	$N_i(x)$	$Q_i(x)$	$M_i(x)$	$N_i(x)$	$Q_i(x)$	$M_i(x)$
1	-51,314	-71,254	-230,773	170,036	48,103	15,707	175,2307	54,0483	28,47
2							-175,2307 175,2307	-36,8918 36,8918	16,1521 -16,1521
3				-170,036 170,036	-14,312 14,312	45,913 -45,913	-175,2307 175,2307	-19,3801 19,3801	43,8247 -43,8247
4							-175,2307 175,2307	-1,6806 1,6806	54,3812 -54,3812
5	51,324 -51,324	98,384 -38,384	45,539 -45,539	-170,036 114,603	20,059 2,902	43,221 -43,221	-175,2307 119,7979	16,0382 6,9228	47,7562 -47,7562
6							-119,7979 119,7979	5,3238 -5,3238	49,1509 -49,1509

7				-114,603 114,603	20,411 -20,411	31,719 -31,719	-119,7979 119,7979	17,5261 -17,5261	38,7167 -38,7167
8							-119,7979 119,7979	29,568 -29,568	16,5089 -16,5089
9	36,647 -36,647	59,021 0,979	-84,628 84,628	-114,603 72,177	43,067 -0,64	-23,449 23,449	-119,7979 77,3715	41,3344 1,092	-17,3481 17,3481
10							-77,3715 77,3715	6,3313 -6,3313	-19,0456 19,0456
11				-72,177 72,177	14,31 -14,31	-32,257 32,257	-77,3715 77,3715	13,7017 -13,7017	-28,116 28,116
12							-77,3715 77,3715	20,9469 -20,9469	-44,5122 44,5122
13	28,63 -28,63	-0,402 60,402	-123,621 123,621	-72,177 49,216	27,52 27,913	-70,192 70,192	-77,3715 54,4105	27,9956 27,4372	-68,1418 68,1418
14							-54,4105 54,4105	-22,6393 22,6393	-43,4428 43,4428
15				-49,216 49,216	-19,154 19,154	-24,962 24,962	-54,4105 54,4105	-17,6235 17,6235	-23,7802 23,7802
16							-54,4105 54,4105	-12,4398 12,4398	-9,271 9,271
17	49,566 -49,566	-66,963 96,963	0 0	-49,216 49,216	-9,705 39,705	0 0	-54,4105 54,4105	-7,1393 37,1393	0 0
18							-54,3013 54,3013	-31,639 31,639	31,4059 -31,4059
19				-58,797 58,797	-29,506 29,506	51,239 -51,239	-54,3013 54,3013	-25,8676 25,8676	57,3608 -57,3608
20							-54,3013 54,3013	-19,8687 19,8687	77,7358 -77,7358
21	-234,416 234,416	-220,416 220,416	140,191 -140,191	-58,797 58,797	-16,607 16,607	82,603 -82,603	-54,3013 54,3013	-13,6975 13,6975	92,4287 -92,4287
22							-54,3013 54,3013	-7,4102 7,4102	101,3684 -101,3684
23				-58,797 58,797	-3,294 3,294	93,388 -93,388	-54,3013 54,3013	-1,0635 1,0635	104,5149 -104,5149
24							-54,3013 54,3013	5,2856 -5,2856	101,859 -101,859
25	-221,787 221,787	-203,782 203,782	217,341 -217,341	-58,797 58,797	10,034 -10,034	84,136 -84,136	-54,3013 54,3013	11,5802 -11,5802	93,4224 -93,4224
26							-54,3013 54,3013	17,7637 -17,7637	79,2571 -79,2571
27				-58,797 58,797	22,983 -22,983	54,77 -54,77	-54,3013 54,3013	23,7804 -23,7804	59,4459 -59,4459
28							-54,3013 54,3013	29,5752 -29,5752	34,1021 -34,1021
29	-242,071 242,071	-225,447 225,447	36,815 -36,815	-58,797 58,797	35,149 -35,149	4,611 -4,611	-54,3013 54,3013	35,0946 -35,0946	3,3696 -3,3696
30							-54,3013 54,3013	40,2864 -40,2864	-32,5768 32,5768
31				-58,797 58,797	46,111 -46,111	-67,569 67,569	-54,3013 54,3013	45,1005 -45,1005	-73,5315 73,5315
32							-54,3013 54,3013	49,4887 -49,4887	-119,258 119,258
33	-246,679	-300,922	-566,265	-58,797	55,42	-163,443	-54,3013	53,4056	-169,4885

The forces of the element clamped-joint, outline into circumference are determined at two means – as a reactive forces at joints in the local coordinate system of the element and as a general expression, known from the FEM.

The results from the solved examples are compare with the results, obtained with an exact expressions for the stiffness matrix of the used tip elements. The comparisons show that the results, for the general displacements from the approximation solution are come nearly to the exat. The results for forces from the approximation solution are come slowly to the exact. For two groups quantities, with different discretization steps are looking a differences in the results (especially at smaller number elements) and a conguance to the exact solution.

CONCLUSIONS

The investigation stiffness matrix of the element clamped-joint give a possibility to use the element for the analysis of structures with joints.

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Докладът е рецензиран.