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Equations of motion of two-phase turbulent flows

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***Abstract**: In current work is making a conclusion of the equations for two-phase turbulent flow in observance of the two-fluid scheme. The hypothesis which is used is that each phase is a separate fluid medium and it is describes with its own (respective) system of differential equations. It is given the type of characteristic equation and value of parameter in it..*

***Key words:** equations of motion, two-phase flow*

INTRODUCTION

In many of our developments over the past quarter century is used the so-called two-fluid scheme of two-phase flow [1,2,3,4]. For the completeness this hypothesis will be formulated here.

TWO PHASE FLOW. TWO FLUID SCHEME OF THE FLOW

It is assuming that [5,6] both phases (can also be leveraged successfully and for multiphase flows) is a separate fluid medium, which is characterized by its own velocity, temperature and density. The two fluid medium are described by the same type and structure systems of partial differential equations of Newtonian type. The relationship between these systems of equations is the forces of interfacial interaction. It is making the following assumptions:

1. The carrier medium generates motion, respectively through its amount of movement provides transportation of impurities. Moreover, the forces of interfacial interactions are recorded with the sign "-" in the equations of the carrier medium and a "+" sign in impurities.
2. Phase of impurities does not posses its own internal stress tensor but has tensor of turbulent stresses ie It has its own different from the carrier medium turbulence. This means that it not take into account the pressure and the viscosity. $\left(p=0;\upsilon=0\right)$
3. The phase of impurities is regarded as "non solid" fluid medium so it is not taking into account strokes between particles and also the exchange of an amount of movement between the particles (droplets) impurities is not take into account.
4. For the phase of impurities has continuity condition of the medium, formulated on the basis of their volume concentration and particle size. This condition is different from the condition of the carrier medium and the minimum amount of computational cell in numerical calculations and experimental research distance between the points in which are measured the parameters.

MATHEMATICAL MODEL

Studies have been made on the basis of two-fluid model based on the equations of Reynolds at a fixed model of turbulence. To provide a choice of different models of turbulence while maintaining a common characteristic equation requires the use of a new approach at conclusion for the equations of motion. The resulting system of equations is similar to the Navier-Stokes equations, which are used as a starting point in their conclusion. In their solution they need only to determine the turbulent viscosity on the basis of the selected model of turbulence. The last one requires the creation of additional computational program for its determination.

As output for the conclusion of the equations of motion of turbulent two-phase flow are used the Navier-Stokes equations, where μ has been replaced by $\mu_{ef}=\mu+\mu_t$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho X-\frac{1}{\rho}\frac{\partial p}{\partial x}+\mu_{ef}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\mu_{ef}}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \quad (1)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho Y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\mu_{ef}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\frac{\mu_{ef}}{3}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \quad (2)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho Z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\mu_{ef}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\frac{\mu_{ef}}{3}\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right) \quad (3)$$

In the phase of the carrier medium whose parameters are identify with subscript "g" - $\mu_{ef}=\mu+\mu_t$. Phase of impurities are marked by "p" - $\mu_{ef}=\mu_t$, $p=0$. This is a consequence from condition which are accepted the above that the phase of impurities does not have its own internal stress tensor.

On the right side of eq. (1 ÷ 3) introduced the components of the vector $\vec{F}$ - field intensity of body forces:

$$\vec{F}=X_{\vec{i}}+Y_{\vec{j}}+Z_{\vec{k}} \quad (4)$$

The connecting link between the two systems of equations of motion phases are the forces of interfacial interaction. With F_x, F_y, F_z are denoted forces or components acting along the axes x, y, z. These forces or their components are recorded with"-" sign in the equations of the carrier phase and with "+" sign in this phase of impurities. It is assumed that the turbulent dynamic viscosity of the two phases is described by different laws i.e:

$$\mu_{tg}\neq\mu_{tp}. \quad (5)$$

In this approaches the equations of motion of the phases have type:

- For gas phase

$$\rho_g\frac{\partial u_g}{\partial t}+\rho_g u_g\frac{\partial u_g}{\partial x}+\rho_g v_g\frac{\partial u_g}{\partial y}+\rho_g w_g\frac{\partial u_g}{\partial z}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u_g}{\partial x^2}+\frac{\partial^2 u_g}{\partial y^2}+\frac{\partial^2 u_g}{\partial z^2}\right)+$$
$$+\frac{1}{3}\mu\frac{\partial}{\partial x}\left(\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)+\mu_{tg}\left(\frac{\partial^2 u_g}{\partial x^2}+\frac{\partial^2 u_g}{\partial y^2}+\frac{\partial^2 u_g}{\partial z^2}\right)+\frac{1}{3}\mu_{tg}\frac{\partial}{\partial x}\left(\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)-F_x \quad (6)$$

$$\rho_g\frac{\partial v_g}{\partial t}+\rho_g u_g\frac{\partial v_g}{\partial x}+\rho_g v_g\frac{\partial v_g}{\partial y}+\rho_g w_g\frac{\partial v_g}{\partial z}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_g}{\partial x^2}+\frac{\partial^2 v_g}{\partial y^2}+\frac{\partial^2 v_g}{\partial z^2}\right)+$$
$$+\frac{1}{3}\mu\frac{\partial}{\partial y}\left(\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)+\mu_{tg}\left(\frac{\partial^2 v_g}{\partial x^2}+\frac{\partial^2 v_g}{\partial y^2}+\frac{\partial^2 v_g}{\partial z^2}\right)+\frac{1}{3}\mu_{tg}\frac{\partial}{\partial y}\left(\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)-F_y \quad 7)$$

$$\rho_g\frac{\partial w_g}{\partial t}+\rho_g u_g\frac{\partial w_g}{\partial x}+\rho_g v_g\frac{\partial w_g}{\partial y}+\rho_g w_g\frac{\partial w_g}{\partial z}=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w_g}{\partial x^2}+\frac{\partial^2 w_g}{\partial y^2}+\frac{\partial^2 w_g}{\partial z^2}\right)+$$
$$+\frac{1}{3}\mu\frac{\partial}{\partial z}\left(\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)+\mu_{tg}\left(\frac{\partial^2 w_g}{\partial x^2}+\frac{\partial^2 w_g}{\partial y^2}+\frac{\partial^2 w_g}{\partial z^2}\right)+\frac{1}{3}\mu_{tg}\frac{\partial}{\partial z}\left(\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)-F_z \quad (8)$$

$$\frac{\partial}{\partial x}\left(y^j\rho_g u_g\right)+\frac{\partial}{\partial y}\left(y^j\rho_g v_g\right)+\frac{\partial}{\partial z}\left(y^j\rho_g w_g\right)=0 \quad (9)$$

- for the phase of impurity $\left(p=0;\upsilon=0;\mu=0\right)$

$$\rho_p \frac{\partial u_p}{\partial t} + \rho_p u_p \frac{\partial u_p}{\partial x} + \rho_p v_p \frac{\partial u_p}{\partial y} + \rho_p w_p \frac{\partial u_p}{\partial z} = \mu_{tp}\left(\frac{\partial^2 u_p}{\partial x^2} + \frac{\partial^2 u_p}{\partial y^2} + \frac{\partial^2 u_p}{\partial z^2}\right) + \\ + \frac{1}{3}\mu_{tp}\frac{\partial}{\partial x}\left(\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} + \frac{\partial w_p}{\partial z}\right) - F_x \quad 10)$$

$$\rho_p \frac{\partial v_p}{\partial t} + \rho_p u_p \frac{\partial v_p}{\partial x} + \rho_p v_p \frac{\partial v_p}{\partial y} + \rho_p w_p \frac{\partial v_p}{\partial z} = \mu_{tp}\left(\frac{\partial^2 v_p}{\partial x^2} + \frac{\partial^2 v_p}{\partial y^2} + \frac{\partial^2 v_p}{\partial z^2}\right) + \\ + \frac{1}{3}\mu_{tp}\frac{\partial}{\partial y}\left(\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} + \frac{\partial w_p}{\partial z}\right) - F_y \quad 11)$$

$$\rho_p \frac{\partial w_p}{\partial t} + \rho_p u_p \frac{\partial w_p}{\partial x} + \rho_p v_p \frac{\partial w_p}{\partial y} + \rho_p w_p \frac{\partial w_p}{\partial z} = \mu_{tp}\left(\frac{\partial^2 w_p}{\partial x^2} + \frac{\partial^2 w_p}{\partial y^2} + \frac{\partial^2 w_p}{\partial z^2}\right) + \\ + \frac{1}{3}\mu_{tp}\frac{\partial}{\partial z}\left(\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} + \frac{\partial w_p}{\partial z}\right) - F_z \quad 12)$$

$$\frac{\partial}{\partial x}\left(y^j \rho_p u_p\right) + \frac{\partial}{\partial y}\left(y^j \rho_p v_p\right) + \frac{\partial}{\partial z}\left(y^j \rho_p w_p\right) = 0 \quad (13)$$

Eq (9) and (13) are the equations of continuity of the phases.

Heat transfer equations for both phases and interfacial Heat is including Q_j have type:

$$\rho c_{pg}\left(\frac{\partial T_g}{\partial t} + u_g \frac{\partial T_g}{\partial x} + v_g \frac{\partial T_g}{\partial y} + w_g \frac{\partial T_g}{\partial z}\right) = \frac{\partial}{\partial x}\left[\left(\lambda_g + \lambda_{Tg} + \lambda ng\right)\frac{\partial T_g}{\partial x}\right] + \\ + \frac{\partial}{\partial y}\left[\left(\lambda_g + \lambda_{Tg} + \lambda ng\right)\frac{\partial T_g}{\partial y}\right] + \frac{\partial}{\partial z}\left[\left(\lambda_g + \lambda_{Tg} + \lambda ng\right)\frac{\partial T_g}{\partial z}\right] + q_{vg} \pm Q_j \quad (14)$$

$$\rho c_{pp}\left(\frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} + w_p \frac{\partial T_p}{\partial z}\right) = \frac{\partial}{\partial x}\left[\left(\lambda_p + \lambda_{Tp} + \lambda np\right)\frac{\partial T_p}{\partial x}\right] + \\ + \frac{\partial}{\partial y}\left[\left(\lambda_g + \lambda_{Tp} + \lambda np\right)\frac{\partial T_p}{\partial y}\right] + \frac{\partial}{\partial z}\left[\left(\lambda_p + \lambda_{Tp} + \lambda np\right)\frac{\partial T_p}{\partial z}\right] + q_{vp} \pm Q_j \quad (15)$$

where: c_{pg}, c_{pp} - specific heat capacity at $p = const.$ for carrier phase and phase of impurities; λ_g, λ_p - thermal conductivity; $\lambda_{Tg}, \lambda_{Tp}$ - coefficient of turbulent conductivity; $\lambda_{rg}, \lambda_{rp}$- radiant heat transfer coefficient for both phases; q_{vg}, q_{vp} - intensity of possible internal sources of heat carrier phase impurities; Q_j - interfacial heat transfer.

At above equations are added the equation of the state of the carrier phase.

$$p = \rho_g R T_g \quad (16)$$

The system equations (6÷15) describes the movement of two-phase non-isothermal non-steady turbulent flow. It's most difficult for solving case, but also the most complete record of the laws of motion of such flow.

CHARACTERISTIC EQUATION

Eq. (6÷15) can be reduced to a common characteristic equation which can be solved numerically.

Φ	Γ	S
1	0	0
u_g	$\mu+\mu_{tg}$	$\Gamma\left[\frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)\right]-F_x-\frac{\partial p}{\partial x}$
v_g	$\mu+\mu_{tg}$	$\Gamma\left[\frac{1}{3}\frac{\partial}{\partial y}\left(\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)\right]-F_y-\frac{\partial p}{\partial y}$
w_g	$\mu+\mu_{tg}$	$\Gamma\left[\frac{1}{3}\frac{\partial}{\partial z}\left(\frac{\partial u_g}{\partial x}+\frac{\partial v_g}{\partial y}+\frac{\partial w_g}{\partial z}\right)\right]-F_z-\frac{\partial p}{\partial z}$
u_p	μ_{tp}	$\Gamma\left[\frac{1}{3}\frac{\partial}{\partial x}\left(\frac{\partial u_p}{\partial x}+\frac{\partial v_p}{\partial y}+\frac{\partial w_p}{\partial z}\right)\right]-F_x$
v_p	μ_{tp}	$\Gamma\left[\frac{1}{3}\frac{\partial}{\partial y}\left(\frac{\partial u_p}{\partial x}+\frac{\partial v_p}{\partial y}+\frac{\partial w_p}{\partial z}\right)\right]-F_y$
w_p	μ_{tp}	$\Gamma\left[\frac{1}{3}\frac{\partial}{\partial z}\left(\frac{\partial u_p}{\partial x}+\frac{\partial v_p}{\partial y}+\frac{\partial w_p}{\partial z}\right)\right]-F_z$
T_g	$\lambda_p+\lambda_{Tg}+\lambda_{rg}$	$q_{vg}\pm Q_j$
T_p	$\lambda_p+\lambda_{Tp}+\lambda_{rp}$	$q_{vp}\pm Q_j$

Characteristic equation for solving the system of equations (6 ÷ 15) has the form:

$$\frac{\partial}{\partial t}(\rho\Phi)+div(\rho V_k\Phi)=div(\Gamma grad\Phi)+S \qquad (17)$$

Where Φ a dependent variable; Γ - the diffusion coefficient of Φ, S - source article index k means "g" for the gas phase and the "p" for phase impurities.
Values Γ and S for each Φ are given in Table 1

The solution of the characteristic equation (17) is made by the method of finite differences. For this purpose it is necessary to make relevant program with which successively are determinates values of Φ at a certain point of the fluid space.

Subroutines are needed to calculate the coefficient of turbulent viscosity of the two phases. Each turbulent model is requires such programs as in choosing his current operator is turns to him. The choice of the model of turbulence is dictated by the particular problem which will be solved.

CONCLUSION

In the work is made a conclusion of equations of turbulent two-phase fluid using a two-fluid scheme of the flow. The resulting system of equations for the two phases, connected to each other by the forces of interfacial interaction is differ from the equations of Reynolds and on its outer structure are similar to those of the Navier-Stokes. As such, they allow for broad, unlimited choice of models of turbulence. This makes their use considerably acceptable perspective. There is more consistent with the physical nature of the investigated processes using the most suitable for solution of turbulent models.

Literature

[1] Antonov I.S., N.T.Nam, Two-fluid integral method for numerical investigation of two-phase turbulent jet, VMEI Sofia, v.45, book 3,1990

[2] Elgobashi, S.E., T.W. Abou – Arab, A second – order turbulence model for two-phase flow, Int. J. Heta transfer, 5, 1982, pp 219-224

[3] Elgobashi, S.E., T.W. Abou – Arab,A two – equation turbulence model for two-phase flow, Phys. Fluids, 26, 4, 1983
[4] Mostafa , A.A., H.C. Mongia, V.G. McDonell, G.S. Samelsen, Evolution of Particle – Laden Jet flow, a theor and exper. Study, AiAA J, v.27,2 1989 pp167-183
[5] Shrajber A.A., L.B. Gavin, V.A. Naumov, V. P. Yasenko, Turbulent flows in gas cuurents, Kiev,1987

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