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PROPAEDEUTICS OF LETTERS SYMBOLISM IN TEACHING MATHEMATICS AT PRIMARY SCHOOL

ПРОПЕДЕВТИКА НА БУКВЕНАТА СИМВОЛИКА В ОБУЧЕНИЕТО ПО МАТЕМАТИКА В НАЧАЛНИТЕ КЛАСОВЕ

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***Abstract**: **Some ideas for propaedeutics of the letters in primary school:** In this paper, we present some various meanings of denoting a number with a letter in mathematics and discuss some ideas for propaedeutics of the letters in primary school based on arithmetic problems.*

***Key words:** propaedeutics, primary school, arithmetic problems, denoting a number with a letter*

INTRODUCTION

The subject of mathematics education in the primary grades is focused primarily on the concept of natural number, the basic arithmetic operations addition, subtraction, multiplication and division, some properties of the operations, the concepts of basic geometric shapes (circle, triangle, rectangle) and the related definitions, theorems and problems. Because definitions, theorems and problems are inherently reasonings or logical conclusions (predicates), the building blocks in the methodology of teaching mathematics are methods for utilizing *concepts, reasoning and logical conclusions* (predicates) associated with mathematical objects.

In the scientific work of the famous methodologist Professor D.Sc. Ivan Ganchev can be found the philosophical and methodological foundations of categories, important for the mathematics education, such as *understanding, belief, guessing, propedeutics, analytical and synthetic method, and modeling*. The Professor reveals both the explanatory and structural features of the dialectical law of quantitative accumulations and qualitative leaps in teaching mathematics. In this respect, it is important to note that the formation of the students' skills to operate with abstract symbols is a difficult and lengthy process, and an important and significant place in this process is occupied by identity transformations of algebraic expressions. Given the dialectical law of quantitative accumulations and qualitative leaps and the need for studying school algebra with comprehension, it is clear that there has to be a initial algebraic propedeutics still in primary school. Of course, the first step in this work is denoting the number with a letter.

The need for propaedeutics of algebraic knowledge in the mathematical training of students in the primary grades is conditioned by the fact that in many countries such as Russia, Serbia, Macedonia, Singapore, France, etc., this propaedeutics begins at the initial stage of training (first, second or third grade) with the introduction of denoting the number with a letter, while in Bulgarian school this knowledge has not been taught since 2002. The same is planned for the school mathematics curriculum for fifth grade.

BODY

The introduction of the letter symbols and, in particular, the denoting of the natural number with a letter from the Latin alphabet and the formation of the ability to use this knowledge in different situations, are important components of propaedeutics of algebraic knowledge in primary school.

The propaedeutics of letter symbolism is expressed most often in:

- replacing a number with a letter;
- recording the properties of the arithmetic operations addition, subtraction, multiplication and division (commutative, associative, and a distributive);
- summarizing the relationship between the numerical characteristics of different objects;
- denoting a variable or a parameter in an algebraic record and etc.

Opportunities to introduce letter symbolism in the study of mathematical knowledge in elementary school may appear in various situations. Adequate and appropriate for the perceptive abilities of students in the primary grades didactic situations are discussed below.

1) Summarizing mathematical reasoning

As an example of academic activities, we will offer the case where basic arithmetic facts associated with the properties of arithmetic operations are revealed, for example: $2+5=5+2$; $78+12=12+78$ and so on. Inductive reasoning leads to the inference expressing the validity of this statement for any two random natural numbers. What follows next is the recording in words of the commutative property of addition and the same applies to rationally performing the calculations in numeric expressions. We believe that in this case it would be appropriate and useful, with the assistance of the teacher, to write the summary $a+b=b+a$, where the letters a and b denote random natural numbers.

2) Denoting fixed but unknown numbers with letters

The letters x and y in the equations $2x-3=5$ and $y+5=8$ substitute the numbers for which both equations are correct. When considering equations of this type (such as lesson task or model of a textual task), it is very important that students understand that in the case with the letters x and y are labeled numbers for which these equations are true, i. e. they do not denote random numbers (in this case $x=4$, $y=3$).

3) Expression of functional dependencies

The letters x and y in the equation $y=4.x$ can be replaced with a multitude of integers for which the equation will be true ($x=1, y=4$; $x=2, y=8$; $x=3, y=12$ and etc.). In this case it is necessary to make clear that these are not random numbers, and between them there is a correlation (in this case the value of y is always 4 times greater than the value of x).

4) Use of lettering as a parameter

As an example, we offer the solution of a word problem [2]:

"How many anima legs will there be in the farmyard if each of the animals in the yard has two legs? And how many will their legs be if these animals are quadrupeds? How can you express the number of legs if each animal in this yard has n number of legs?

Solution: If we denote the total number of legs with y, and with x - the number of animals in the yard, the equations $y = 2.x$ and $y = 4.x$ express the dependencies between the number of animals and their legs when they have respectively 2 or 4 legs.

In the case where the number of legs that each animal has is recorded summarily by the letter n, it results in the dependency $y = n.x$.

The problem illustrates the case when the letter n is a parameter in the equation $y = n.x$ and determines the behavior of the function that includes the variables x and y, i.e. the parameter is a quantity whose value determines the characteristics or the behavior of other variables.

Our opinion is that such problems can be solved in extracurricular activities at the end of fourth grade, and also in the training of students with a keen interest in mathematics. This will prepare and implement a "smooth" transition to the study of algebraic knowledge in the higher grades. Indeed, later in the sixth grade in Mathematics Education letters are used as random or abstract symbols in algebraic expressions. In problems such as: "Decompose into multipliers the expression: $m^2 + 6m$," the letter m does not replace a specific number, because it does not solve an equation for a particular value of m. Upon decomposition of the expression $m^2 + 6m$, the letter is an abstract symbol which is manipulated by applying a multitude of rules. In this case the use of a distributive property of the operation of multiplication in relation to the operation of addition allows for the breakdown of the expression into multipliers: $m^2 + 6m = m(m + 6)$.

The existing practice and current observations of students to whom the letter symbolism in solving relevant problems has not been introduced in the primary grades, shows that at a later stage they lack the habits for using letters in modeling of word problems and in writing equations. Such students always strive to solve the problem arithmetically and in some cases they themselves cannot accurately and clearly substantiate and justify the reason for carrying out a series of arithmetic operations. There are no prerequisites for the formation of analytical skills, logical thinking and the development of a mathematical style of reasoning.

We believe that in even in the early classes of mathematics education (3 and 4 grades), on the basis of the school curriculum and the knowledge already acquired, students should be directed to performing some simple and somewhat intuitive deductions. For example, even in primary school summary record for representing any odd number using letter symbols can be derived.

Table 1.

1	1 = 2.0 + 1
3	3 = 2.1 + 1
5	5 = 2.2 + 1
7	7 = 2.3 + 1
9	9 = 2.4 + 1
11	11 = 2.5 + 1
13	13 = 2.6 + 1
15	15 = 2.7 + 1
...	...

The table record shows that any odd number is presented in the form $2n+1$ where n is an integer. Then the students can be asked the question: *"Can you say what number you will get when you add two odd numbers?"*

Solution: if n and m denote two random integers, using the basic properties of the arithmetic operations students can obtain the answer to the question what number is the result when two random odd numbers are added:

$$(2n+1)+(2m+1)=(2n+2m)+(1+1)=2(n+m)+2=2(n+m+1),$$

i.e. to reach the deduction that the sum of any two odd numbers is always an even number.

The modeling of conditions and the solutions of some word problems is an adequate and very appropriate occasion for the introduction of letter symbolism.

"Annie has 3 chocolates. Ina has 4 chocolates more than Annie. How many chocolates does Ina have? "[3]

Solution: This particular problem is a primal problem concerning the operation of addition. [4] The components in the structure of the problem are three, and the first two are known by their numerical characteristics, while the third is unknown. Such problems are solved using addition and their solution can be represented by the following model (Fig. 1):

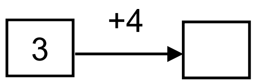

Fig. 1

Although the solution of the problem lies in calculating the numerical expression $3+4=\ldots$, the problem can be modeled with the equation: $3+4=x$, where x denotes the number of Ina's chocolates, i.e. the model of the problem is as follows (Fig. 2):

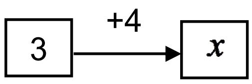

Fig. 2

When students are taught to write equations and to model word problems, it will be easier for them if the arithmetic operation is addition or subtraction of integers. In this sense, the teaching of mathematics in primary school, where most of the educational content is related to the addition and subtraction of integer, is a natural starting point for the introduction and use of letter symbols. Moreover, the modeling of word problems using equations not only helps students to reflect on the role of the letter as fixed but unknown number, but also prepares students to solve algebraic equations in the study of algebra in the upper grades.

The same arithmetic problem may be offered to students after some information from its condition is "erased”:

"Annie has several chocolates. Ina has 4 chocolates more than Annie. How can you record the number of chocolates that Ina has? "

When the task is formulated in this way, students have difficulty in constructing the expression which necessarily requires the introduction of a letter. In this example, the difficulty comes from the fact that the number of chocolates that Annie has is not known and must be marked with a letter. The conducted observations and taking into consideration the experience and skills that students have at this level of their mathematics education, which is focused mainly on their arithmetic training and work with concrete numbers, the first intention and attitude of most of them is to find the number of chocolates that Annie has. The point of giving such a problem in the primary grades is the comprehension and understanding of the idea that the number of chocolates that Annie has is unknown and is intended to remain so, so that the solution to the problem will not be a number but an expression which will contain a letter. Then, if the number of chocolates that Annie has is denoted by n , the problem will the following solution model (Fig. 3):

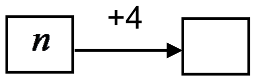

Fig. 3

i.e. Ina's chocolates will be presented with the expression $n+4$.

What distinguishes this particular problem from the problems typical for this level of training is that the solution is not limited to performing calculations with specific numbers, but students are required to compile an expression that corresponds to the imposed condition and this is only possible with the participation of a letter.

Discussing such problems creates prerequisites for the formation of basic concepts related to algebra, where the composing of, operation with and carrying out of reasonings with expressions involving letters are an essential part of algebraic operations. The formation of such knowledge and skills can begin in the primary grades in the context of arithmetic.

CONCLUSION

Acquiring, even in the primary grades, experience for working with letters that perform different roles depending on the situation in which they are used (to summarize dependencies; to replace an unknown number; to indicate a variable) prepares students for real algebra. In the formal study of algebra in subsequent classes, such knowledge is consolidated and deepened as students get acquainted with the role of the letter as a parameter.

The letter symbolism provides an efficient way of expressing algebraic dependencies and of solving many mathematical problems. In this sense, the use of letter symbols is an essential mathematical tool and it is good for its introduction to be carried out gradually, starting as early as primary school, in order to enable students to work with letter symbols and develop the habit for their application even in the first years of their mathematics studies.

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