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ANIMATED DRAWINGS IN THE STUDYING OF TRIGONOMETRIC FUNCTIONS IN THE SCHOOL MATHS LESSONS

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Animated drawings in the studying of trigonometric functions in the school Maths lessons: *One of the key problems in the Maths's education is the problem of computer and IT technologies' integration into Trigonometric in the school Maths course. The main purpose of this elaboration is to visualize analytically the knowledge of trigonometric functions (sin, cos, tg, cotg) in order to understand the whole and complete idea for the same. One of the didactical methods to achieve this is to use an Animated drawings and Interactive geometrical system GeoGebra. To illustrate better the ideas, there are some real examples from the class experience.*

Keywords: *Circle. Trigonometric function, Animated drawings, Interactive geometrical system GeoGebra*

ABSTRACT

Methods of mathematics teaching take a central place in the didactics and methods of school teaching. The didactically beneficial combination between "traditional" and "modern" teaching methods in the education process and seeking methods to clarify the linguistic presentation and logic of mathematics are the most important tasks of the mathematics teacher.

It is well-known that very frequently mathematics is understood as a combination of ready and established deductive theories and that they are "a steady form that never moves" [4]. In reality, this is not the case. Abstract mathematical categories – concepts, assertions and inferences – are not learned but formed (built) in the mind of the learner. This means that the starting point is a lower degree of abstraction, i.e. in a sense the idea is "broken down" to reach the mind of the learner and then be "constructed" again. But a process can be adequately reflected only through another process. In other words, mathematical knowledge is formed only by its "**visualization"** [4]. It is not proper and useful when the teacher only informs learners of the new knowledge. This knowledge must be represented by the teacher so as to reach the mind of learners. The fast development of computer and information technologies and their application in all spheres of public life, including in the sphere of education, give broad opportunities to transform teaching mathematics at school into **visualization** of mathematics.

One of the means of visual presentation of mathematical knowledge as a process (creation, development and application) is computer animation using the *GeoGebra* dynamic education software.

RESULTS AND DISCUSSION

Practice has shown that successful learning of trigonometric functions at school requires the creation of special situations in the learning process and giving students opportunities to obtain from them obvious patterns, geometric facts, ideas for proof, etc. So frequent use is made of animated drawings based on geometric modeling of individual operations and their presentation as animated objects. The main advantage of this teaching approach over classical methods is that

it gives opportunities to present animated drawings modeling continuously changing geometric figures and graphs of functions. It is known from practice that animation use brings movement in teaching and recreation of mathematical knowledge, in particular trigonometric knowledge. Animation shows various properties of trigonometric functions (increasing, decreasing, maximum, minimum, etc.) and their changes resulting from change in one or more of the conditions. Moreover, watching changing drawings (graphs of functions) students can analyze, infer and find function properties, formulate hypotheses and try to explain them with methods they already know. So it is appropriate to say that teaching trigonometric functions with animated drawings at school is a modern empiric approach achieving different didactic purposes:

"**Bringing to life**" concepts of trigonometric functions (*sin(x), cos(x), tg(x), cotg(x)*) and their properties by means of previously prepared by the teacher applets allowing students to watch concepts in the process of their "development" and making them realize on their own ideas and understand the presented school material;

Formation of **basis** for developing logical thinking in students by the teachers who, while developing geometric models and graphs of trigonometric functions, must show every step of drawing; this will enable them to ask students questions and provoke them to build hypotheses, deny or prove these hypotheses and reach correct conclusions;

Purposeful **"translation"** from analytical recording of trigonometric functions in the language of graphs and by means of this development of "reading" skills for graphs of functions, understanding and accuracy in running graph algorithms.

Our ideas for use of animated graphs through the *GeoGebra* education software in teaching trigonometric functions in the school mathematics course will be illustrated by two examples from the teaching practice of the article author – Nikolinka Bachvarova, mathematics teacher in School of Mathematics Acad. K. Popov, Plovdiv.

1. Use of animated graphs as a means of gaining new knowledge

An important moment requiring animation in the study of trigonometric functions is the change of position of a point from a single circle (depending on the angle) and the simultaneous projection и (building of the image и) on the graph of the respective trigonometric function. We will present the building of the graph of the function *sin(x)*, recorded with the equation $y = sin(x)$.

A graph of the function $y = sin(x)$ is built in the interval $x \in [0; 2\pi]$.

In the "static" illustration the single circle with centre (-1;0) is divided into 12 equal parts and the definition of the function $y = sin(x)$ is used to build its graph. This results in central angles with coordinates $\alpha=0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\ldots 2\pi$. The relevant points P_α of the single circle are marked and straight lines parallel to the x-axis are built "through" them [3]. This results in points obtained from the crossing of these lines with the lines $x = \alpha$. Connecting all obtained points we get the graph of the function $y = sin(x)$.

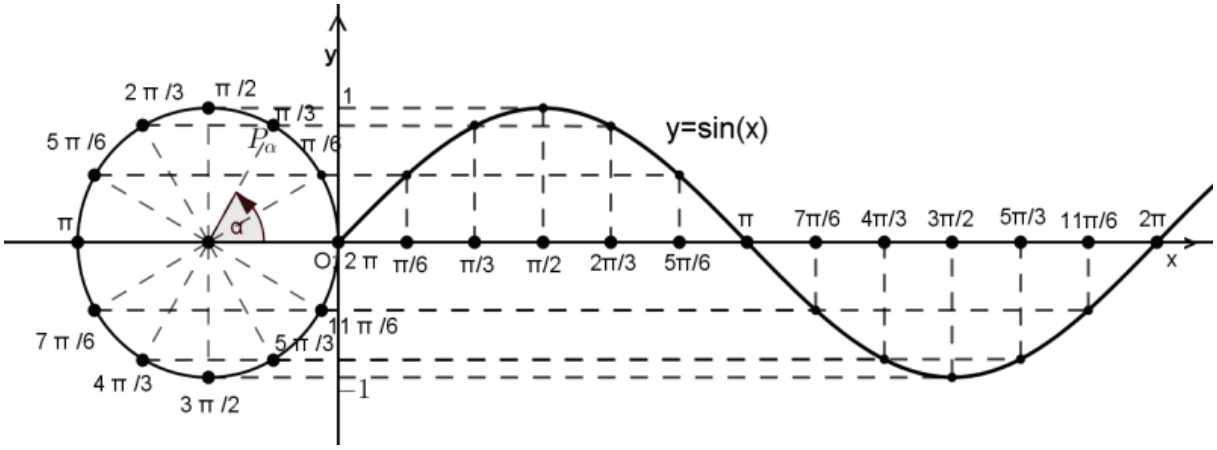

Fig. 1

This process of drawing the function graph $y = sin(x)$ can be presented as animated drawing ([1], [7]) with the successive tracing of each point on the graph. The animated drawings process may be accelerated, slowed down or even temporarily stopped for detailed explanations by the teacher.

To continue drawing on the graph (on the Ox axis) the periodic function property must be used.

The dynamic drawing (fig.1) allows observation of the building graph process in the method proposed above with the option to fix unlimited number of points and view the graph obtained in the interval $[0;2\pi]$.

Fig.2 shows animated drawing of the same function and the angle is given as a parameter realized through a scroll bar. In the animation the point moves along the sine wave and shows dependency of position ŷ on angle size.

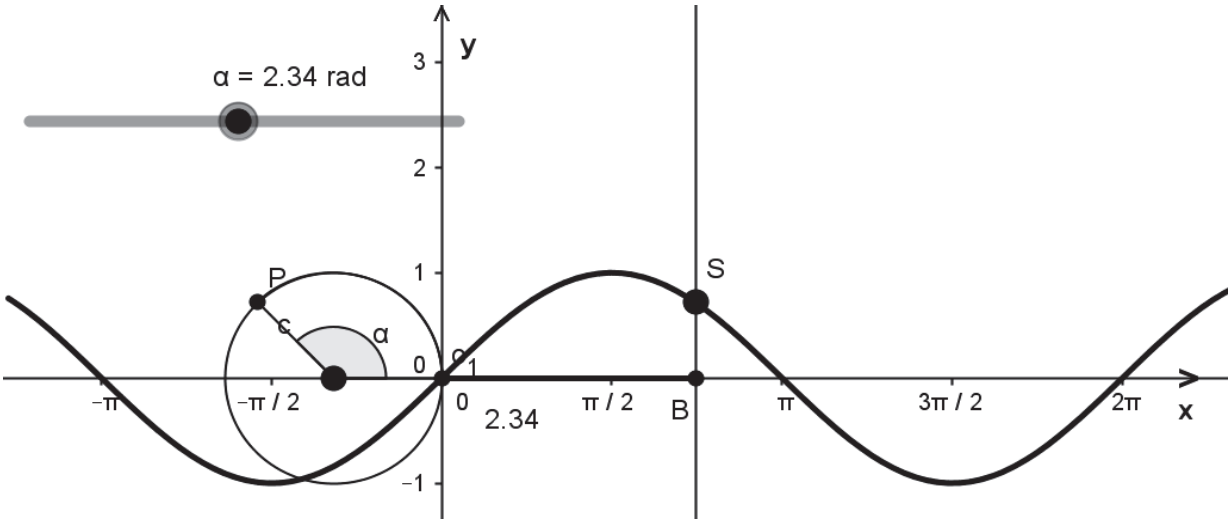

Fig. 2

Analogously, it is possible to show animated graphs of the other trigonometric functions [7]. However, for the functions $y = tg(x)$ and $y = cotg(x)$ the animation can be made by two scroll bars – for change of angle in the interval $[0^\circ; 360^\circ]$ and the interval $[-360^\circ; 0^\circ]$. These scroll bars are "parameters" of the angles of reviewed intervals. In the drawing process every graph point leaves a trace visualizing obtained graphs.

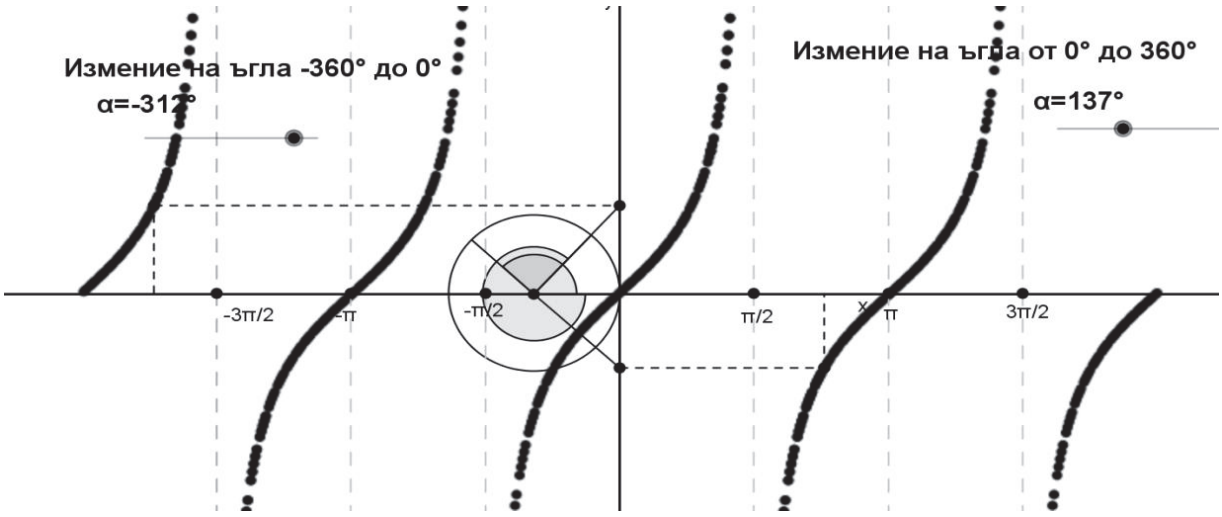

Fig. 3

2. Using animated drawings as a means of stimulating student research

To allow students to reach a conclusion and formulate a hypothesis related to finding properties or algorithms of operation while exploring a mathematical object it is necessary to review various single cases, to repeat different actions multiple times, to compare and analyze the results from these actions. Animated drawings with *GeoGebra* make it possible to build this high number of single objects for a very short time interval and it apparently seems that the object is only one. With its "movement" in the animated drawing various results are obtained and they form the basis of correct conclusions. The advantage of applying animated drawings is that the test may be of frontal type, i.e. the teacher asks the whole class questions. Analyzing obtained results, students are provoked to build hypotheses whose correctness must be proved or denied.

The dynamic software *GeoGebra* may be successfully applied for building graphs of complex functions by using geometric transformations of graphs of elementary functions which are already studied. E.g. after the function $y = f(x)$ is reviewed and the building of its graph is taught, the following functions may be analogously reviewed:

$$y = f(x) + k;\ y = f(x + k);\ y = k.f(x),$$

where κ is a parameter.

(A similar experiment is made while teaching the topic "*Properties of trigonometric functions of extended angle*" [2] to students of 10^3 form at School of Mathematics Acad. K. Popov, Plovdiv where animated drawings are used.)

First, a graph is built of the elementary function $y = sin(x)$. Building graphs of composite (complex) functions ($y = \kappa.\ sin(x)$, $y = sin(x) + \kappa$, $y = sin(x + \kappa)$) is performed by parameters realized in *GeoGebra* by scroll bar or parameter in the function definition.

Fig. 4 presents a demo realized with parameters in the definitions of used functions. In it, check buttons are used to select the properties tested by students. This allows the teacher to choose which properties to demonstrate and which results to compare, generalize and respectively check. When choosing the respective function, next to the check button appears an explanatory text and symbolic visualization related to a dynamic text so that after changing the position of the point from the main sine wave this results in a change in screen inscription.

Fig. 4

After changing the position of the point, the graph is "transformed" (widening, shortening, symmetry, etc.). Comparing the obtained image with the graph of the elementary function $y = sin(x)$, students can find the type and position of the graph of complex functions against the coordinate system and against the graph of the function $y = sin(x)$. Observing the dynamics of "transformations", students may understand better the properties of trigonometric functions of

extended angle: periodicity, parity, imparity, amplitude, symmetry, etc. In this case the education software *GeoGebra* is used not only to visualize the study of trigonometric functions and show beautiful applets but also to provide an opportunity to students to show maximum activity and independence while finding function properties, to encourage their logical thinking, to make conclusions and inferences. Of course, good results can also be achieved after properly asked questions by the teacher and after well animated demonstrations.

CONCLUSION

Animated drawings may sucessfully be used in trigonometry in the solution of trigonometric equations and inequalities with parameters in the solution of problems of applied type, in problems including geometric point locations, etc.

Personal experience in school practice use of animated drawings with *GeoGebra* suggests the following generalized conclusions:

1. Information and computer technology change the nature of teaching activities with new didactic approaches for reaching set education goals

2. There is optimization of the lesson teaching process and more effective use of timing stages

3. Animated drawings can be used in different stages of the teaching process – introduction of new knowledge, gaining knowledge, control and assessment, etc.

4. Animated drawings gives dynamic visualization and "relation" between specific visual and abstract logical thinking as well as opportunities for "transition" from one to the other type of thinking.

5. Study of trigonometric functions with animated drawings is the starting point for making hypotheses and justification of formulated statements.

In conclusion, proper use of computer animation in mathematics teaching depends on the teacher and the teacher's decision when and how much animated drawings will be used in the teaching process.

REFERENCES

[1] Dimkova D., Study and Teach Mathematics with *GeoGebra*, Vezni-4, Sofia, 2012 p. 29.

[2] Dodunekov St et al., Mathematics for the 10th grade, Specialized Edition, Regalia 6, Sofia, 2007, p. 132 – 135.

[3] Dodunekov St. Et al., Mathematics for the 11th grade, Specialized edition, Regalia 6, Sofia, 2002, p. 128 – 134.

[4] Lalchev, Z.M.Varbanova. Teaching and Visualization of mathematics in the Teaching of Student-Pedagogues. In: Collection of Fifth Autumn Scientific Conference, Kiten, 2007, p. 260 - 264.

[5] Larin S.V., Collection of Materials from IV International Scientific methodological Conference, Kransoyarsk, 18-19 November 2015, p. 38

[6] URL: http://www.geogebra.org

[7] URL: http://www.vivacognita.bg

