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AN ALGORITHM FOR SYNNTHESIS OF BINARI NEARLI PERFECT SIGNALS

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Abstract: *The phase manipulated (PM) radio signals witch ideal auto – correlation function (ACF), resembling the Dirac delta – function, are very important for radars, radio-navigation and radio-synchronization systems, because they provide the maximal possible resolution of the objects. With regard to this fact, in the paper a new algorithm for synthesis of binary PM signals with nearly ideal periodic ACF is suggested. The computational effectiveness of the algorithm is demonstrated by several examples of unknown up to now binary nearly perfect signals, synthesised by the respective computer program.*

Keywords: *computer algorithm, digital signal processing, binary nearly perfect signals*

INTRODUCTION

The phase manipulated (PM) radio signals witch ideal auto – correlation function (ACF), resembling the Dirac delta – function, are very important for some types radio communication systems such as: radars, radio-navigation and radio-synchronization systems. Due to this reason they have been intensively researched during the past seventy years [3], [5], [7], [8], [9]. Despite of the all taken efforts only four methods for synthesis of signals with ideal ACF are found. According to the names of the inventors they can be classified as methods of: Frank-Zadoff-Heimiller, Chu, Milewski and Ipatov [1], [2], [4]. At the moment these signals cannot meet rapidly growing expectations of the designers and engineers. With regard to this situation, in the paper we propose a new algorithm for synthesis of binary PM signals with nearly ideal periodic ACF (PACF) and present the results, obtained by an universal computer program, developed on its base.

The paper is organized as follows. First, the mathematical model of the binary PM signals with ideal PACF is recalled. After that a new algorithm for synthesis of binary nearly perfect PM signals is suggested. At the end the computational effectiveness of the algorithm is demonstrated by several examples of unknown up to now binary nearly perfect PM signals, synthesised by the computer program, developed on its base.

MATHEMATICAL MODEL OF THE BINARY PHASE MANIPULATED SIGNALS WITH IDEAL PERIODIC AUTO-CORRELATION FUNCTION

As known [3], [5], [9], [10] the PM signals with ideal PACF are very important for radars, radio-navigation and radio-synchronization systems, because they provide the maximal possible resolution of the objects. Besides, in the wireless communication systems these signals diminish the negative effects, caused by the multipath spread of the electromagnetic waves and by simultaneous asynchronous communication of multiple users. Due to their positive features the PM signals with ideal PACF are named *perfect signals* or *sequences* and are intensively researched during the past seventy years [3], [5], [9], [10].

As mentioned above, the PACFs of perfect PM signals resemble the Dirac delta - function (pulse). This means that their PACF does not possess any side-lobes:

$$q_{\zeta\zeta}(r)=\sum_{i=0}^{N-1}\zeta(i).\zeta^*\langle i+r\rangle_N=\begin{cases}N, & r=0,\\ 0, & r\neq 0.\end{cases} \qquad (1)$$

In (1) $q_{\zeta\zeta}(r)$ is the discrete PACF of the perfect PM signal at discrete time-shift *r*, the symbol "$\langle\ \rangle_N$" means "summation modulo *N*", and symbol "*" – "complex conjugation". Besides, in (1) it is used the fact that every periodic PM signal with period $T_r = N\tau_{ch}$ is a sequence (row - vector):

$$\{\zeta(i)\}_{i=0}^{N-1}=\{\zeta(0),\zeta(1),...,\zeta(N-1)\}, \qquad (2)$$

consisting of the complex numbers $\zeta(0),\zeta(1),...,\zeta(N-1)$, which are the complex envelopes of the elementary phase chips with duration τ_{ch}, forming the PM signal. They are generated by the rule:

$$\zeta(i)=\exp\left[j\frac{2\pi l}{M}u(i)\right],\quad j=\sqrt{-1},\ i=0,1,...,N-1 \qquad (3)$$

In (3) $l,\ 1\leq l<M$ is arbitrary integer co-prime with the size *M* of the phase constellation (or alphabet), $u(i)$ are the elements of the integer sequence, controlling the initial phases of the chips:

$$\{u(i)\}_{i=0}^{N-1}=\{u(0),u(1),...,u(N-1)\},\quad \forall u(i)\in\{0,1,...,M-1\}. \qquad (4)$$

The sequence (4) is called *characteristic signal* of the PM signal (1).

The binary PM signals are the most preferable from the implementation point of view, because it is easy to generate and to process the chips of the signals, as the signal constellation (alphabet) consists of only $M=2$ elements: $+1$ and -1 [3], [5], [9], [10].

Unfortunately during the past seventy years only one binary perfect signal was found, which is the Barker sequence of length $N=4$ [3], [5], [9]:

$$\{u(i)\}_{i=0}^{3}=\{+1,+1,-1,+1\}. \qquad (5)$$

Due to the total lack of binary perfect signals a great practical importance has the exploration of the binary nearly perfect signals, which PACF has as small number of side-lobes with small magnitudes as possibly. This problem is studied in more detail in the next part of the paper.

AN ALGORITHM FOR SYNTHESIS OF BINARY NEARLY PERFECT SIGNALS

In the theory of synthesis of periodic binary PM signals it is proven that [3], [5], [9]

$$q_{\zeta\zeta}(r) \equiv N \bmod 4 . \tag{6}$$

Consequently, only PM signals with lengths

$$N \equiv 0 \bmod 4 \tag{7}$$

can have small number of side-lobes. From the fact that PACF of the binary PM signals is symmetric to the side-lobe $q_{\xi\xi}(N/2)$

$$q_{\xi\xi}(r) = q_{\xi\xi}(N-r),\ 1 \le r < N/2, \tag{8}$$

it can be concluded that the binary PM signals, which PACF is

$$q_{\zeta\zeta}(r) = \sum_{i=0}^{N-1} \zeta(i).\zeta^*\langle i+r \rangle_N = \begin{cases} N, & r=0; \\ \pm 4, & r = N/2; \\ 0, r \notin \{0, N/2\} \end{cases} \tag{9}$$

are the best possible approximation of the binary perfect signals. Due to this reason the binary PM signals with PACF (9) should be considered as the best examples of binary nearly perfect signals.

In order to obtain necessary conditions (or sieves) for reducing the computational complexity in the synthesis of nearly perfect signals, the PACF of a PM signal will be presented in the following algebraic form:

$$\left\langle F_\zeta(x).F_\zeta^*(x^{-1}) \right\rangle_{\bmod (x^N-1)} = q_{\zeta\zeta}(0) + q_{\zeta\zeta}(1)x + q_{\zeta\zeta}(2)x^2 + \ldots + q_{\zeta\zeta}(N-1)x^{N-1} \tag{10}$$

Here the symbol "$\langle\ \rangle_{\bmod(x^N-1)}$" shows that all the powers $x^i, i = 0,1,\ldots$ of the indeterminate x are reduced modulo $x^N - 1$. Besides, $F_\zeta(x)$ is the so-named Hall polynomial associated with the PM signal (2). Its coefficients are the samples of the PM signal (2)

$$F_\zeta(x) = \zeta(0) + \zeta(1)x + \ldots + \zeta(N-1)x^{N-1}. \tag{11}$$

Analogously, the right side of (10)

$$Q_{\zeta\zeta}(x) = q_{\zeta\zeta}(0) + q_{\zeta\zeta}(1)x + q_{\zeta\zeta}(2)x^2 + \ldots + q_{\zeta\zeta}(N-1)x^{N-1} = N \pm 4x^{N/2} \tag{12}$$

is the Hall polynomial of the PACF of the PM signal (2).

As

$$x^N - 1 = \left(x^L - 1\right)\left(x^{(k-1)L} + x^{(k-2)L} + \ldots + x^L + 1\right) \tag{13}$$

for every possible factorization $N = kL$ of *N*, all polynomial congruencies

$$F_\zeta(x).F_\zeta^*(x^{-1}) = q_{\zeta\zeta}(0) + q_{\zeta\zeta}(1)x + \ldots + q_{\zeta\zeta}(N-1)x^{N-1} \ \bmod (x^L - 1) \tag{14}$$

follow from (10).

In the considered case $N \equiv 0 \bmod 4$ the set of factors *L* of *N* is $\{1,2,4\}$. From this fact follow the congruencies

$$[S(0)+S(1)+S(2)+S(3)]^2 = S_Q \mod(x-1), \tag{15}$$

$$\{[S(0)+S(2)]+[S(1)+S(3)]x\}\{[S(0)+S(2)]+[S(1)+S(3)]x^{-1}\} = N \pm 4 \mod(x^2-1), \tag{16}$$

$$\begin{aligned}&[S(0)+S(1)x+S(2)x^2+S(3)x^3][S(0)+S(1)x^{-1}+S(2)x^{-2}+S(3)x^{-3}] = \\ &= N \pm 4 \mod(x^4-1),\ N \equiv 0 \bmod 8\end{aligned}, \tag{17}$$

$$\begin{aligned}&[S(0)+S(1)x+S(2)x^2+S(3)x^3][S(0)+S(1)x^{-1}+S(2)x^{-2}+S(3)x^{-3}] = \\ &= N \pm 4x^2 \mod(x^4-1),\ N \equiv 4 \bmod 8\end{aligned} \tag{18}$$

Here $S(0), S(1), S(2), S(3)$ and S_Q stand for:

$$\begin{aligned}&S(0) = \zeta(0)+\zeta(4)+\ldots+\zeta(N-4),\ S(1) = \zeta(1)+\zeta(5)+\ldots+\zeta(N-3),\\ &S(2) = \zeta(2)+\zeta(6)+\ldots+\zeta(N-2),\ S(3) = \zeta(3)+\zeta(7)+\ldots+\zeta(N-1),\\ &S_Q = q_{\zeta\zeta}(0)+q_{\zeta\zeta}(1)+q_{\zeta\zeta}(2)+\ldots+q_{\zeta\zeta}(N-1).\end{aligned} \tag{19}$$

In the considered case PACF has only one side-lobe $q_{\xi\xi}(N/2) = \pm 4$ and due to this reason the left side of (15) is $S_Q = N \pm 4$.

The polynomial congruency (16) is equivalent to the following system of equations

$$\left|\begin{aligned}&[S(0)+S(2)]^2+[S(1)+S(3)]^2 = N \pm 4\\ &2[S(0)+S(2)][S(1)+S(3)] = 0\end{aligned}\right. \tag{20}$$

Here it should be pointed out that the following transformations do not change the PACFs of PM signals [3], [5], [7], [8]:

(T1) Cyclical shift of the PM signal (2) at $1, 2, \ldots, N-1$ positions;

(T2) Inversion of the signs of the all samples; in other words the PACFs of the PM signal (2) and the PM signal $\{-\zeta(i)\}_{i=0}^{N-1} = \{-\zeta(0), -\zeta(1), \ldots, -\zeta(N-1)\}$ coincide;

(T3) Alternation of the signs of the all samples; this means that the PACFs of the PM signal (2) and the PM signal $\{(-1)^i \zeta(i)\}_{i=0}^{N-1} = \{\zeta(0), -\zeta(1), \zeta(2), -\zeta(3), \ldots, -\zeta(N-1)\}$ coincide.

With these transformations every PM signal can be converted into another PM signal with the same PACF so that the following inequalities to be fulfilled:

$$\begin{aligned}&S(0)+S(1)+S(2)+S(3) \geq 0,\\ &S(0)+S(2) \geq S(1)+S(3) \geq 0,\\ &S(0) \geq S(2).\end{aligned} \tag{21}$$

Besides, the sums (19) must satisfy the restrictions

$$S(0), S(1), S(2), S(3) \in \{-N/4, -N/4+2, -N/4+4, \ldots, N/2-2, N/2\}, \tag{22}$$

which follow from the fact that the signal constellation (alphabet) consists of only $M = 2$ elements: $+1$ and -1. More specifically, it is easy to prove that the sum *S* of even (odd) quantity *K* of $+1$s and -1s is an even (odd) integer in the range $[-K, K]$.

On the base of (15), (20) and (21) the following necessary conditions for existence of binary nearly perfect signals can be extracted:

(NC1) The integer $N \pm 4$ must be a perfect square of the form $[S(0)+S(2)]^2 = N \pm 4$ and

$$S(0)+S(2) = \sqrt{N \pm 4}\,; \tag{23}$$

(NC2) The sum of the samples with odd indices of the PM signal must be zero

$$S(1)+S(3)=0. \tag{24}$$

The polynomial congruency (17) is equivalent to the following system of equations

$$\left|\begin{array}{c} S^2(0)+S^2(1)+S^2(2)+S^2(3)=S_Q \\ S(0)S(1)+S(1)S(2)+S(2)S(3)+S(3)S(0)=0 \\ 2[S(0)S(2)+S(1)S(3)]=0 \end{array}\right. \tag{25}$$

Analogously, the polynomial congruency (18) is equivalent to the following system of equations

$$\left|\begin{array}{c} S^2(0)+S^2(1)+S^2(2)+S^2(3)=N \\ S(0)S(1)+S(1)S(2)+S(2)S(3)+S(3)S(0)=0 \\ 2[S(0)S(2)+S(1)S(3)]=\pm 4 \end{array}\right. \tag{26}$$

As

$$S(0)S(1)+S(1)S(2)+S(2)S(3)+S(3)S(0)=[S(0)+S(2)][S(1)+S(3)] \tag{27}$$

the second equations of the systems (20), (25) and (26) coincide. Due to this reason from (25) and (26) only two additional necessary conditions can be extracted:

(NC3) The integers *N* and $S_Q = N \pm 4$ must be sum of four perfect squares

$$S^2(0)+S^2(1)+S^2(2)+S^2(3)=\begin{cases} N\pm 4, & N\equiv 0 \bmod 8; \\ N, & N\equiv 4 \bmod 8; \end{cases} \tag{28}$$

(NC4) The sums (19) of the samples of the PM signal must satisfy the equalities

$$S(0)S(2)+S(1)S(3)=\begin{cases} 0, & N\equiv 0 \bmod 8; \\ \pm 2, & N\equiv 4 \bmod 8; \end{cases} \tag{29}$$

The restrictions (21) and (22) and the necessary conditions (23), (24), (28) and (29) leads to the following algorithm.

Algorithm for synthesis of binary nearly perfect signals

Step 1. Choosing of the length $N\equiv 0 \bmod 4$ of the PM signal so that $S_Q = N \pm 4$ to be a perfect square;

Step 2. Generating of the set of all vector-rows $\{S(0), S(1), S(2), S(3)\}$ in accordance with the restrictions (21), (22) and the necessary conditions (23), (24), (28) and (29);

Step 3. Transforming of the every vector-row $\{S(0), S(1), S(2), S(3)\}$, obtained at the *Step* 2, in the corresponding vector-row

$$\begin{gathered} S(k)=\sum_{i=0}^{N_4-1}\zeta(4i+k),\ k=0,1,2,3,\ N_4=N/4,\ \Rightarrow \\ \{\zeta(4i+k)\}_{i=0}^{N_4-1}=\{\zeta(k),\zeta(4+k),\ldots,\zeta(4(N_4-1)+k)\},\forall\zeta(i)\in\{-1,+1\}, \end{gathered} \tag{30}$$

Step 4. Selecting of binary PM signals $\{\zeta(i)\}_{i=0}^{N-1}=\{\zeta(0),\zeta(1),\ldots,\zeta(N-1)\}$, obtained at the *Step* 3, which PACF satisfy (9).

The above *Algorithm for synthesis of binary nearly perfect signals* was realized as an universal program in Matlab environment. Only three classes of non-equivalent binary nearly perfect signals were found with it during an exhaustive investigation for lengths $N \le 50$. They are presented in Table I, where for brevity only the set $I_{\sup}$ of positions of elements $\zeta(i) = -1$ are shown.

Table I. Binary nearly perfect signals with lengths $N \le 50$, which PACFs have one side-lobe

No	**N**	$S_Q = N \pm 4$	$I_{\sup}$	γ
1	4	0	$\{2,3\}$	∞
2	8	4	$\{1,2,4\}$	1,3333
3	12	16	$\{1,2,4,8\}$	1,1250

Besides, the losses γ in the *signal-to-noise ratio* (SNR) after processing with *side-lobes suppression filter* (SLSF), were estimated. Here it should be recalled that side-lobes of PACFs of the PM signals can be eliminated by an appropriate mismatched filter, named SLSF. Unfortunately, this positive effect is accompanied by losses in the SNR. They are measured by the so-named coefficient of losses in the SNR [5], [6], [7], [8], [10]

$$\gamma = \sum_{k=0}^{N-1} \frac{1}{\left|F_{\zeta}(w^k)\right|^2} \tag{31}$$

Here $F_{\zeta}(w^k)$, $k = 0,1,...,N-1$ is the discrete Fourier spectrum of the processing PM signal. It can be easily evaluated by the substitutions $x = w^k$, $w = \exp(j\frac{2\pi}{N})$, $k = 0,1,...,N-1$ in the Hall polynomial of the PM signal. It is proven that the lowest bound of the coefficient of losses is 1 [6].

The above algorithm was modified for exploration of binary nearly perfect signals which PACFs have 2, 3, 4 and 5 side-lobes. It was realized also as an universal program in Matlab environment. With it many non-equivalent binary nearly perfect signals were found during an exhaustive investigation for lengths $N \le 50$. They are presented in Table II, where against for brevity only the set $I_{\sup}$ of positions of elements $\zeta(i) = -1$ are shown.

Table II. Binary nearly perfect signals with lengths $N \le 50$, which PACFs have 2, 3, 4 and 5 side-lobes

No	**N**	S_Q	$I_{\sup}$	γ
1	16	16	$\{0,1,2,4,8,13\}$	1,2589
2	20	16	$\{0,1,2,3,5,8,12,16\}$	1,1750
3	20	36	$\{0,1,2,4,8,11,16\}$	1,1111
4	24	16	$\{0,1,6,9,11,15,18,19,22,23\}$	1,0607
5	24	36	$\{2,3,5,8,14,18,19,20,22\}$	1,0667
6	28	16	$\{0,1,5,7,9,10,11,14,19,22,26,27\}$	1,1673
7	28	36	$\{0,1,3,5,6,7,10,13,18,19,27\}$	1,1305
8	32	16	$\{1,2,3,4,6,10,13,15,20,21,23,29,30,31\}$	1,1536

9	32	36	$\{0,1,2,3,5,6,9,15,17,19,22,26,27\}$	1,0950
10	36	36	$\{1,2,4,7,8,9,11,15,17,18,19,20,23,27,32\}$	1,0455
11	40	36	$\{1,2,4,5,10,13,17,18,19,20,23,24,25,28,30,32,34\}$	1,0575

The obtained results, presented in Table I and Table II, show that the minimal number of side-lobes grows slowly with the increasing of the length *N* of the binary nearly perfect signals. Despite of this fact, the coefficient of losses in the SNR, caused by the processing with SLSF, is small and decreases with the increasing of the length *N*. Consequently, the binary nearly perfect signals in fact compensate the lack of binary perfect signals and they can be successfully applied in the present radars, radio-navigation and radio-synchronization systems.

CONCLUSIONS AND FUTURE WORK

In the paper a new algorithm for synthesis of binary nearly perfect signals is suggested. It is effective from computational point of view and provides synthesis of binary nearly perfect signals with following positive features:

1) Lengths $N \equiv 0 \bmod 4$ of signals form an infinite set, which ensures:

- flexibility in the process of development of new radars, radio-navigation and radio-synchronization systems;
- small spectral density, which makes the binary nearly perfect signals practically invisible for the radio-electronic intelligence;

2) Possibility to eliminate all the side-lobes of the real PACF with SLSF, operating with very small losses in the SNR.

The presented in the paper algorithm and universal computer program for synthesis of binary nearly perfect signals could be useful in the modelling and developing of new radars, radio-navigation and radio-synchronization systems, providing:

- the maximal possible resolution of the objects;
- diminishing of the negative effects, caused by the multipath spread of the electromagnetic waves and by simultaneous asynchronous communication of multiple users.

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