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# THE PHYSICAL MECHANIZM OF REDUCING FIELD CHARGES DEPOSITED ON DIELECTRIC

## Assoc. Prof. Gulizar Alisoy, PhD

Department of Mathematics, Namik Kemal University, Tekirdag, Turkey Phone: (+90)282 2502734 E-mail: galisoy@nku.edu.tr

**Prof. Hafiz Alisoy, DcS** Department of Electronics and Communication Engineering, Namik Kemal University, Tekirdag, Turkey Tel.: (+90)282 2502386 E-mail: halisoy@nku.edu.tr

**Abstract:** In this study, the reasons leading to the weakening of the fields of charges settled on the dielectric are examined. It should be noted that the representation of the gap model using the equivalent circuit, as well as the values of  $U_{ig}$  and  $U_{ex}$ , is a rather rough approximation to reality.

On the basis of an analysis of the two-dimensional equivalent circuit of the discharge gap using the Green's function method for two cases: (i) The centrally symmetric distribution in the form of a normal Gaussian distribution; (ii) the periodic distribution of the density of free charges, the causes of the weakening of the field of charges on the dielectric are analysed.

Keywords: Green's function method, Dielectric, Model, Surface charge.

## **INTRODUCTION**

In the case of a real breakdown of an elementary segment of the gap, charges with an uneven density settle on opposite sides of the gap. Because of this, the surface of the dielectric after one of the elementary discharges cannot be considered an equipotential surface, because of which the values of  $U_{ig}$  and  $U_{ex}$  lose their meaning. The field in the gap bounded by the dielectric does not remain homogeneous after the first discharge. In addition to the constituent of the normal surface of the dielectric, a tangential component of the electric field strength arises, due to the nonuniform density of charges deposited on the dielectric. This component of the field leads to charge mixing over the surface of the dielectric due to the presence of surface conductivity. In addition, the density of charges can decrease due to the bulk conductivity of the dielectric. The desire to reflect the influence of the surface and volume conductivities of the dielectric on the discharge processes in the gap led to the complication of the equivalent circuit (Kuchinskii 1979, Grigoryev 2006).

#### THEORY

Let us consider the influence of the volume and surface conductivities of a dielectric on the process of decreasing the surface density of charges that have settled on a dielectric (see Fig1). We assume that the dielectric layer by the thickness is  $d_d$  located on an unlimited metal plane. Its bulk and surface conductivities are  $\gamma$  and  $\gamma_s$ . On the dielectric surface, a nonuniform distribution of free charges  $\sigma(x, y)$  is given. The potential of both metal surfaces is assumed to be zero. If the length at which the value of the density of surface charges significantly changes significantly more  $d_d$  and d, then the described system can be considered as consisting of separate elementary capacitances with leakages proportional to  $\gamma$  and connected by conductances proportional to  $\gamma_s$ . (Bortnik & et al. 1999, Blennow & et al.2000, Alisoy & et al. 2005). Thus, in this approximation our analysis will be similar to the analysis of the two-dimensional equivalent circuit of the discharge gap.



**Figure1**. Two-dimensional equivalent circuit of the discharge gap by considering surface conductivity of dielectric

The potential of the surface of the dielectric  $\varphi(x, y)$  can be written in the form

$$\varphi(x,y) = \frac{\sigma(x,y)}{c_d + c_g} \tag{1}$$

where  $C^* = C_d + C_g$  is the sum of the specific capacitances of the dielectric and gas layers. The change in the density of surface charges will be due to both the surface current and the current flowing in the bulk of the dielectric. In this case, to change the charge density, we can write

$$\frac{d\sigma}{dt} = -divj_s - \frac{\gamma}{d_d}U \tag{2}$$

Taking into account that the surface current will be determined by the potential gradient over the surface, that is,  $J_s = -\gamma_s grad U$  for the potential on the surface of the dielectric, we obtain the following equation

$$\frac{\partial\varphi}{\partial t} + a\varphi = \mathbf{b}\nabla^2\varphi \tag{3}$$

where  $a = \frac{\gamma}{d_d C^*}$   $\mu b = \frac{\gamma_s}{C^*}$ .

The expression (3), with the help of the substitution  $\varphi(x, y, t) = \Phi(x, y, t)exp(-at)$ , reduces to the form

$$\frac{\partial U}{\partial t} = b\nabla^2 U \tag{4}$$

The required form of the potential on the dielectric surface can be obtained by solving equation (4) with the initial condition

$$U(x, y, 0) = \frac{\sigma(x, y)}{c_d + c_g}$$

As can be seen, equation (4) is a two-dimensional heat equation, and to solve it we apply the method of Green's functions. We take the two-dimensional Green's function in the form (Tikhonov & Samarskii, 2013)

$$G(x, y, \xi, \eta, t) = \frac{1}{4\pi bt} exp\left[-\frac{(x-\xi)^2 + (y-\eta)^2}{4bt}\right]$$
(5)

where  $\xi$  and  $\eta$  are the coordinates of the instantaneous sources, respectively. In this case the solution of equation (4) will have the form

$$U(x, y, t) = \int_{-\infty}^{+\infty} U(\xi, \eta, 0) G(x, y, \xi, \eta, t) d\xi d\eta$$
(6)

#### ANALYSIS OF THE RESULTS.

We consider two special cases. Suppose that the distribution of the density of free charges on the surface of a dielectric has the form of a normal Gaussian distribution. Then the initial density of charges is defined as follows

$$\sigma(r) = \sigma_0 exp\left[-\left(\frac{r}{r_0}\right)^2\right] \tag{7}$$

This case corresponds physically to the spreading of the accumulation of charges due to a single solitary discharge channel. The solution of the statement problem has the form

$$\varphi(r,t) = \frac{\sigma_0}{C^*} \frac{r_0^2}{4bt + r_0^2} exp\left[-at - \frac{r^2}{4bt + r_0^2}\right]$$
(8)

From this it is clear that the change in the potential of the dielectric surface is not subject to an exponential law. In this case in the center of the distribution, the potential decreases as

$$\varphi(0,t) = \frac{\sigma_0}{c^*} \frac{r_0^2}{4bt + r_0^2} exp(-at)$$
(9)

Now let the distribution of the density of free charges on the surface of a dielectric be of the form of a periodic distribution with initial charge density as

$$\sigma(x,y) = \frac{\sigma_0}{2} (1 + \cos\omega x \cos\omega y) \tag{10}$$

In this case,  $\varphi(x, y, 0)$  is a potential relief in the form of alternation of protuberances and troughs with period A along the x and y axes. This case corresponds physically to the spreading of the accumulation of charges, when there are many independent discharge channels in the discharge gap.

Thus for the initial time is chosen such that the discharge has already occurred on all these independent sections of the dielectric.

Taking the expression for the Green's function in the form (5), we obtain the solution

$$U(x, y, t) = \frac{\sigma_0}{8\pi bC^*} [S(x)S(y) + S_1(x)S_1(y)]$$
(11a)

where

$$S(x) = \int_{-\infty}^{\infty} exp\left[-\frac{(x-\xi)^2}{4bt}\right] \cdot \cos\left(\frac{2\pi\xi}{A}\right) d\xi$$
(11b)

$$S(y) = \int_{-\infty}^{\infty} exp\left[-\frac{(y-\eta)^2}{4bt}\right] \cdot \cos\left(\frac{2\pi\eta}{A}\right) d\eta$$
(11c)

$$S_1(x) = \int_{-\infty}^{\infty} exp\left[-\frac{(x-\xi)^2}{4bt}\right] d\xi$$
(11d)

$$S_1(y) = \int_{-\infty}^{\infty} exp\left[-\frac{(y-\eta)^2}{4bt}\right] d\eta$$
(11e)

Calculating the integrals, we obtain the solution of the statement problem as

$$\varphi(x, y, t) = \frac{\sigma_0}{2C^*} \left[ exp\left(-\frac{8\pi^2 b}{A^2}t\right) cos\omega x \cdot cos\omega y + 1 \right] exp(-at)$$
(12)

The loss of the potential at the center of one of the individual charge clusters (for example, x = 0 and y = 0) has the form

$$\varphi(0,0,t) = \frac{\sigma_0}{2C^*} \left[ exp\left(-\frac{8\pi^2 b}{A^2}t\right) + 1 \right] exp(-at)$$
(13)

In order to be able to compare the results obtained with the experimental data, we replace the processes  $\varphi(0, t)$  and  $\varphi(0,0, t)$  with exponential ones with the same rate of potential change at time t = 0. Calculating the time constant from

$$\tau = -\frac{1}{\varphi} \frac{d\varphi}{dt} \Big|_{t=0}$$
(14)

we obtain for the time constant the following expressions

$$\tau_1 = \frac{4\gamma_s d_d + \gamma r_0^2}{C^* d_d r_0^2}$$
(15a)

$$\tau_2 = \frac{4\pi^2 \gamma_s d_d + \gamma A^2}{C^* d_d A^2} \tag{15a}$$

If the radius of an individual cluster in (10) is taken from the relation  $\pi r_0 = A$ , then the expressions for  $\tau_1$  and  $\tau_2$  will coincide

$$\tau = \frac{4\gamma_s d_d + \gamma r_0^2}{C^* d_d r_0^2} \tag{16}$$



Figure 2. Reduction of the relative density of charges on the dielectric surface as a result of spreading with time with a Gaussian initial charge distribution

The results of calculating the relative charge density for a Gaussian initial charge distribution are shown in Fig. 2. In this case, the values of the potential calculated on the dielectric surface

calculated according to expressions (8) and (12). We note that similar results are obtained also in the case when the distribution of the density of free charges on the dielectric surface has the form of a periodic distribution. As can be seen (see Fig. 2), the amplitude of the distribution of the density of the settled charges during the spreading process changes more rapidly than its radius. In both cases, with an unlimited increase in time, the potential of the dielectric surface tends to zero. This is due to the presence of bulk conductivity of the dielectric.

# CONCLUSION

An analysis of the expressions obtained indicates that the spreading of charges on the dielectric surface is the main reason for the decrease in the field of the settling charges. This, in turn, explains the causes of the experimentally observed oscillations of the discharge current in the discharge gap.

It is shown that the amplitude of the density distribution of the settled charges during the spreading process changes faster than its radius. In both cases considered, with an unlimited increase in time, the potential of the dielectric surface tends to zero. This is due to the presence of bulk conductivity of the dielectric.

At times of the order of, the surface conductivity of the dielectric plays the predominant role, because of which the  $\tau^{-1}$  charge density will be smoothed out at all points of the surface.

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