# INTEGRATION OF INFORMATION TECHNOLOGIES IN THE SOLUTION OF INDICATIVE AND LOGARITHMIC EQUATIONS 

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#### Abstract

The work presents a research in the capacity of MuPAD system to do computer computations in Mathematics training. Tasks, suitable for symbol computer computations, are presented. The work is suitable for extracurricular activity with students who have interest in the computer technology, as well as the organization of Computer mathematics competitions for children.


Keywords: Information technology, Indicative equations, Logarithmic equations, Effectiveness, MuPAD, CAS (Computer Algebra System).

## INTRODUCTION

The development of the modern school in the conditions of information technologies, in the dynamic process of globalization, influences all educational systems. The dynamic development of science and technology leads to the need to change and improve the existing teaching and learning methods. Lifelong learning gains great importance for the success of each person, because it changes the style of work, communication and the way of acquiring competencies. The way the information is presented to students is constantly modernized and this necessitates the search for new tools, forms and methods for updating the educational materials in accordance with society's new requirements of the modern school - to develop the cognitive activity of the students, to create skills for self-replenishing knowledge through different styles and pace of learning, individual interests, motivation, personal quests and creativity.

At the end of 2006, the European Union Parliament adopted recommendations on key competences for lifelong learning. Within the European Reference Framework for key competences is that the young European today must possess mathematical competence and key competences in the natural sciences as well as digital competence (Information Technology). In response to these recommendations, in the autumn of 2006, Bulgaria began to implement an educational project of great importance for all who are connected in some way with the school as well as for the society as a whole. Mandatory learning of modern information technologies began. Today Bulgaria is one of the few countries in Europe that introduces the study of the subject "Information technologies" from $1^{\text {st }}$ to $10^{\text {th }}$ grade.

The change in the educational system raises the need for innovation. On one hand, using new methods and techniques in the educational process in order to provoke students' desire to learn, and on the other - the learning to become more interesting, more engaging and more effective activity.

This paper explores the computer algebra system MuPAD in support of the mathematics training during solving indicative and logarithmic equations.

## EXPOSITION

MuPAD is a modern computer algebra system (CAS) designed to create and operate symbolic objects and is suitable for use in math classes.

The original version of MuPAD was developed at the University of Paderborn, Germany. Since 1997, it has been developed by the company SciFace Sotware jointly with the Paderborn Group. Until 2005 MuPAD Light is distributed for free, then only the paid version of MuPAD Pro is available. In September 2008, SciFace was purchased from The Mathworks Inc, and MuPAD was
embedded in the Symbolic Math Toolbox package of MATLAB. Since 2008 MuPAD ceases to exist as a stand-alone product.

The Symbolic Math Toolbox package is designed to perform symbolic transformations in the MATLAB environment. Until version R2007b inclusive, this package used the Maple kernel for this purpose - one of the most popular computer algebra systems. In the following versions, Maple was replaced with the MuPAD system, which became an integral part of the Symbolic Math Toolbox (Yordanov, Y., 2011).

The universal MuPAD system is oriented towards a wide range of mathematical tasks - it analytically solves complex algebraic and transcendent equations, inequalities and systems; finds derivatives and integrals; there are included specialized subprograms for solving analytical geometry tasks, linear algebra, numerical theory, combinatorics, numerical approximation, linear optimization (simplex method), probability theory and statistics.

MuPAD has the following capabilities (Yordanov, Y., 2011):

- symbolic translations of expressions;
- analytical and numerical solution of equations;
- numerical calculations with simple and arbitrary accuracy;
- superb two-dimensional, three-dimensional graphics and animation;
- programming language, supporting functional and object-oriented programming;
- dialogue and programming mode of operation;
- program packages on linear algebra, numerical methods, statistics, functional programming, etc .;
- using $\mathrm{C}++$ procedures to speed up calculations.


## Using MuPAD to solve indicative and logarithmic equations

To solve the tasks presented in the paper, MuPAD 5.6.0 is used.
Solving equations of the type $\mathrm{a}^{\mathrm{f}(\mathrm{x})}=\mathrm{b}$ and $\log _{\mathrm{a}} \mathrm{f}(\mathrm{x})=\mathrm{b}$ with MuPAD can be done in the following two ways:

1. Using the solve function;
2. Using the plot command (graphically).
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Problem 1. Solve the equation (10 5-x }\mp@subsup{)}{}{6-x}=100\mathrm{ (Lozanov, Ch., Et al., 2005).
Solution.
First method:
solve((10^(5-x) )^(6-x)=100, x, Real)
    {4,7}
```

where in the solve command is first recorded the equation, then the variable toward which the equation is to be solved and the type of variable (real, integer, etc.) is indicated at the end.

## Second method:

plot $\left(\left(10^{\wedge}(5-x)\right)^{\wedge}(6-x)-100, x=3.9 . .7 .5\right)$


Fig. 1. Graphic representation for solving task 1
In the plot command, the function that we want to plot is recorded first, then the drawing interval is specified.

Problem 2. Find the values of the parameter $m$ for which the equation $9^{x}-m \cdot 3^{x}+2=0$ has exactly two real solutions.

## Solution.

Replace $3^{x}=y, y>0$ is put and as a result is derived the square equation $y^{2}-m \cdot y+2=0$, whose discriminant is $D=m^{2}-8$ and the roots are $y_{1}=\frac{m+\sqrt{m^{2}-8}}{2}$ and $y_{2}=\frac{m-\sqrt{m^{2}-8}}{2}$. The equation has real roots if $\mathrm{m}^{2}-8>0 \Rightarrow \mathrm{~m} \in(-\infty ;-2 \sqrt{2}) \cup(2 \sqrt{2} ;+\infty)$. As $\mathrm{y}_{2}<\mathrm{y}_{1}$, then $\mathrm{y}_{2}>0$ (from the assumption), i.e. $\frac{m-\sqrt{m^{2}-8}}{2}>0$ and then $m>\sqrt{m^{2}-8}$. If $\forall \mathrm{m}<0$, i.e. $\mathrm{m} \in(-\infty ;-2 \sqrt{2})$, then $\mathrm{m}<\sqrt{\mathrm{m}^{2}-8}, \mathrm{y}_{2}<0$. Then the equation does not have two real roots.

Therefore, within the range $\mathrm{m} \in(2 \sqrt{2} ;+\infty)$ the equation has exactly two real solutions.
We will present the examined cases graphically.
At $\mathrm{m}=-3$ there are no real solutions:

```
plot(9^x-(-3)* *^x+2, x=-2..2)
```



Fig. 2. Graphic representation of the solution of the equation
At $m=3$ the equation has real solutions:
$\operatorname{plot}\left(9^{\wedge} x-3 * 3^{\wedge} x+2, x=-0.25 \ldots 0.7\right)$


Fig. 3. Graphic representation of a solution of the equation
Problem 3. Solve the equation $\log _{5}\left(\mathrm{x}^{2}-2 \mathrm{x}-2\right)=0$ (Lozanov, Ch., Et al., 2005).

## Solution.

```
solve(log(5, x^2-2*x-2)=0, x, Real)
    {-1,3}
plot(log(5, x^2-2*x-2),x=-1.1..3.1)
```



Fig. 4. Graphic representation of the solution of the equation
Note: We'll look at how the chart changes if the equation is solved $\log _{5}\left(x^{2}-2 x-2\right)=3$.
$p \operatorname{lot}\left(\log \left(5, x^{\wedge} 2-2 * x-2\right)-3, x=-18 \ldots 18\right)$


Fig. 5. Graphic representation of the solution of the equation
Problem 4. Find the values of the real parameter $m$ for which the equation $\log _{2}^{2} \mathrm{x}+\mathrm{mlog}_{2} \mathrm{x}-2 \mathrm{~m}=0$ has a solution in the range $(1 ;+\infty)$ (64th National Olympiad in Mathematics, 2014).

## Solution.

Replace $\log _{2} \mathrm{x}=\mathrm{y}$, i.e. $2^{\mathrm{y}}=\mathrm{x}>1=2^{0} \Rightarrow \mathrm{y}>0$. Then $\mathrm{y}^{2}+\mathrm{m} . \mathrm{y}-2 \mathrm{~m}=0, \mathrm{D}=\mathrm{m}^{2}+8 \mathrm{~m}$ and then $y_{1}=\frac{-m+\sqrt{m^{2}+8 m}}{2}$ and $y_{2}=\frac{-m-\sqrt{m^{2}+8 m}}{2}$.

On order for the equation to have a solution is necessary that $\mathrm{D}>0 \Rightarrow \mathrm{~m} \in(-\infty ;-8) \cup(0 ;+\infty)$ (at $\mathrm{D}=0 \mathrm{~m}=0, \mathrm{x}=1$ is a solution, and at $\mathrm{m}=-8, \mathrm{x}=16$ is a solution). As $\mathrm{y}_{2}<\mathrm{y}_{1}$ in order to get all the solutions for $m(m \neq 0)$ is necessary that $y_{2}>0$ (by the condition).
$\mathrm{y}_{2}=\frac{-\mathrm{m}-\sqrt{\mathrm{m}^{2}+8 \mathrm{~m}}}{2}>0$
At $\mathrm{m} \in(-\infty ;-8) \mathrm{y}_{2}>0$.
At $\mathrm{m} \in(0 ;+\infty) \mathrm{y}_{2}<0$.
$\mathrm{y}_{1}=\frac{-\mathrm{m}+\sqrt{\mathrm{m}^{2}+8 \mathrm{~m}}}{2}$
At $\mathrm{m} \in(-\infty ;-8) \mathrm{y}_{1}>0$.
At $\mathrm{m} \in(0 ;+\infty) \mathrm{y}_{1}>0$.
Answer: $\mathrm{m} \in(-\infty ;-8] \cup(0 ;+\infty)$
Graphical solutions of the equation for different parameter values are plotted.
At $\mathrm{m}=1$ :

```
plot}(\operatorname{log}(2,x)*\operatorname{log}(2,x)+1*\operatorname{log}(2,x)-2*1, x=0..2.5
```



Fig. 6. Graphic representation of the solution of the equation


Fig. 7. Graphic representation of the solution of the equation


Fig. 8. Graphic representation of the solution of the equation


Fig. 9. Graphic representation of the solution of the equation

## CONCLUSION

At the present stage, the School and the University in Bulgaria, as institutions, become an attractive place for personal development and are the basis for future professional realization due to the continuous penetration of information and computer technologies in the education.

It is appropriate to use information and communication technologies, mathematical education software, multimedia educational presentations, etc. in the various stages and levels of mathematics education. Their integration contributes to the stimulation of the cognitive development and critical
thinking, creates conditions for development of mathematical, communicative, social competences, etc. (Vasileva-Ivanova, R., Velikova, E. 2015). This can increase the efficiency of the education in mathematics and stimulate the student's motivation to learn.

The rapid changes in information technology and the large number of mathematical software (Maple, Mathematica, MatLab, etc.) are a prerequisite for a gradual change in the teaching methods for mathematics.

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