# Analysis of the Sufficiency of the Transportation Vehicles for Timely Service of the Patients in an Emergency Medical Centre 

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Analysis of the Sufficiency of the Transportation Vehicles for Timely Service of the Patients in an Emergency Medical Centre: The following issues are discussed: 1.Lack of finances during the depression. There are no new ambulances; 2. The center purchases new ambulances at a rate $\lambda$.
The states of the system have been described via Kolmogorov's equations. The critical periods until which the system can fulfil its duties have been found. The values of the intensities, with which new transportation vehicles have to be purchased, have been defined. That would ensure the servicing of patients without making them wait in the queue for service.

Key words: Emergency Medical Aid, Transport Services, Random Processes

## INTRODUCTION

The emergency medical care centers have only one goal: timely service of the patients that require medical care [2], [6]. Together with the medical teams, great roles have the transportation vehicles, with which the medical teams reach the patients.

As the emergency medical care is financed by the state budget, the circumstances in the prolonged economic crisis have caused reducing of the budgets for all units, it is necessary to answer the question: how long can we postpone the purchasing of new transportation vehicles for the emergency medical care centers, so that with the minimum waiting time in the service queue the patients receive in time medical care that will save their lives or their condition has been alleviated.

Of interest is the question whether the transportation vehicles (ambulances and other vehicles) are sufficient to ensure the timely service to the patients.

## TASK ONE

We examine the process of exploitation of the transportation vehicles in the car park of the emergency medical care center (EMCC) in the city of Ruse. The general setting of the task will be that the number of available transportation vehicles is $n$ at the time of beginning of the study $t=0$. They all have been in exploitation for a different period of time. The rate of the ambulance rejection (the removal from exploitation) for every one of them is constant and equals $\mu$. This means that the time during which the vehicle is in operation without being rejected is a random variable, distributed by an indicative law with parameter $\mu$.

Subject of study is the random variable $\mathrm{X}(\mathrm{t})$ - number of ambulances that are in exploitation. We will assume that in a period of crisis in the car park at the emergency medical care center no new ambulances are purchased. The graph of the system states is shown in Figure 1.


Fig.1. Graph of the system states
With $s_{i}, i=1,2, \ldots, n$ are indicated the system states. Since in state $s_{i}$ there are $i$ ambulances in operation and each one of them generates a flow of rejections and removal from exploitation at rate $\mu$, therefore the cumulative flow that brings the system from state
$s_{i}$ to state $s_{i-1}$ will be calculated by the formula $\sum_{k=1}^{i} \mu=i . \mu(i=n, n-1, \ldots, 1)$. The system of Kolmogorov equations [3], which describes the quantity of $X(t)$, has the form:

$$
\begin{align*}
& \int \frac{d p_{0}(t)}{d t}=\mu \cdot p_{1}(t) \\
& \left\{\frac{d p_{i}(t)}{d t}=(i+1) \cdot \mu \cdot p_{i+1}(t)-i \cdot \mu \cdot p_{i}(t), \quad(i=1,2, \ldots,(n-1))\right.  \tag{1}\\
& \frac{d p_{n}(t)}{d t}=-n . \mu \cdot p_{n}(t)
\end{align*}
$$

The system of equations can be solved if the following initial conditions are met:

$$
p_{n}(0)=1, \quad p_{i}(0)=0, \quad i=0,1, \ldots,(n-1),
$$

i.e. at the beginning of the study all ambulances in the EMCC's car park are in service.

The solution of the system (1) may be obtained by applying the apparatus of operational calculus:

From the last equation we obtain $F_{n}(x)=\frac{1}{x+n \cdot \mu}$. After applying the inverse Laplace transform we find $p_{n}(t)=e^{-n \mu}$. From the last result for $p_{n}(t)$ follows that the law of distribution of the time the system is in state $s_{n}$ is indicative with a parameter $n . \mu$.

After solving system (3) and applying inverse Laplace transform we obtain the following results (4):

$$
\begin{equation*}
p_{n-1}(t)=n \cdot\left[e^{-(n-1), \mu, t}-e^{-n, \mu, t}\right], \quad p_{n-i}(t)=\frac{n!}{(n-i)!} \cdot \sum_{k=0}^{i} \frac{e^{-(n-k), \mu, t}}{\sum_{l=0}^{i} \frac{1}{k-l} \prod_{h=0}^{i}(k-h)}, i=0,1, \ldots,(n-1) . \tag{4}
\end{equation*}
$$

From earlier studies conducted by us [1] at EMCC Ruse it was determined that the center has 8 ambulances. Then from (4) by substituting successively with $i=0,1, \ldots, 7$ we obtain the following results:

$$
\begin{align*}
& p_{8}(t)=e^{-8 \mu . t}, \quad p_{7}(t)=8 \cdot\left[\frac{e^{-8 . \mu, t}}{-1}+\frac{e^{-7 . \mu, t}}{1}\right], \quad p_{6}(t)=56 \cdot\left[\frac{e^{-8 . \mu, t}}{2}+\frac{e^{-7 . \mu, t}}{-1}+\frac{e^{-6 . \mu, t}}{2}\right], \\
& p_{5}(t)=336 .\left[\frac{e^{-8, \mu, t}}{-6}+\frac{e^{-7, \mu \cdot t}}{2}+\frac{e^{-6, \mu, t}}{-2}+\frac{e^{-5, \mu, t}}{6}\right], p_{4}(t)=1680 .\left[\frac{e^{-8, \mu, t}}{24}+\frac{e^{-7, \mu \cdot t}}{-6}+\frac{e^{-6, \mu, t}}{4}+\frac{e^{-5 . \mu, t}}{-6}+\frac{e^{-4, \mu, t}}{24}\right] \text {, } \\
& p_{3}(t)=6720 .\left[\frac{e^{-8, \mu . t}}{-120}+\frac{e^{-7 . \mu \cdot t}}{24}+\frac{e^{-6 . \mu . t}}{-12}+\frac{e^{-5 . \mu . t}}{-12}+\frac{e^{-4 . \mu . t} t}{-24}+\frac{e^{-3 . \mu . \mu t}}{120}\right] \text {, }  \tag{5}\\
& p_{2}(t)=20160 .\left[\frac{e^{-8, \mu . t}}{720}+\frac{e^{-7, \mu \cdot t}}{-120}+\frac{e^{-6 . \mu, t}}{48}+\frac{e^{-5, \mu \cdot t}}{36}+\frac{e^{-4, \mu, t}}{48}+\frac{e^{-3 . \mu, t}}{-120}+\frac{e^{-2, \mu . t}}{720}\right] \text {, } \\
& p_{1}(t)=40320 .\left[\frac{e^{-8, \mu \cdot t}}{-5040}+\frac{e^{-7, \mu \cdot t}}{720}+\frac{e^{-6, \mu \cdot t}}{-240}+\frac{e^{-5, \mu \cdot t}}{144}+\frac{e^{-4, \mu \cdot t}}{-144}+\frac{e^{-3, \mu \cdot t}}{240}+\frac{e^{-2, \mu \cdot t}}{-720}+\frac{e^{-\mu \cdot t}}{5040}\right] .
\end{align*}
$$

From the provided condition the value of $p_{0}(t)=1-\sum_{i=1}^{8} p_{i}(t)$ is found. The mathematical expectation and dispersion of the random variable $X(t)$ are calculated by the formulae (6) from [7]:

$$
\begin{equation*}
M[X(t)]=\sum_{i=1}^{n} i \cdot p_{i}(t) ; D[X(t)]=M\left[X^{2}(t)\right]-(M[X(t)])^{2}=\sum_{i=1}^{n} i^{2} \cdot p_{i}(t)-\left(\sum_{i=1}^{n} i \cdot p_{i}(t)\right)^{2} \tag{6}
\end{equation*}
$$

Since the period of operation of an ambulance is not more than 15 years, therefore the rate of rejection is $\mu=8.10^{-6}$ ambulances per hour.

## CONCLUSIONS BASED ON THE RESULTS FROM TASK ONE

From the obtained results for the values of the mathematical expectation and the dispersion of the random variable $\mathrm{X}(\mathrm{t})$ we conclude that with the increasing number of hours in operation of the ambulances in the car park at the EMCC, the number of ambulances who are in working condition decreases. If no new ambulances are purchased for the EMCC after 9 years, there will remain no more than 4 ambulances, which will be insufficient for the timely service of the patients. Given our previous studies [4] on providing service to patients without them waiting in the queue, it is necessary that at any given time there will be at least four ambulances in working order available. It is necessary to take into account the fact that the ambulances undergo routine repairs and maintenance [5], so that the previously mentioned four ambulances will be available at any time. It is necessary to have at least two more ambulances in reserve. The conducted studies show that the period of four years of operation of the car park will be critical, when $M[X(t)]=$ 6.067 , i.e. there will be 6 ambulances available at the EMCC, so that at least four of them are in working condition and can ensure promptly, without waiting in the queue, service to the patients.

Moreover, at the beginning of the study not all available ambulances are new, therefore the likely rate of rejections will be higher than the one adopted and the critical minimum number of ambulances will be reached in less than four years.

## TASK TWO

We consider the case when new ambulances are purchased for the car park at the EMCC and that is done at rate $\lambda(t)$. Each ambulance is removed from service after a random time T , distributed by indicative law with parameter $\mu$. We consider a random process $X(t)$ - number of ambulances that are operational at a time $t$. We set the task to find the one-dimensional law of distribution of the random variable $X(t)$. We consider the case where no more than $n$ ambulances can be used at the car park at the EMCC. In that case the processes of termination and generation of the system will be observed [7]. The graph of the system states is shown in Figure 2.


Fig.2. Graph of the system states when the number of vehicles has been limited to $n$
If at time $\mathrm{t}=0$ there are k ambulances $(\mathrm{k}=1,2, \ldots, \mathrm{n})$ in the car park, then the initial conditions will be in the form $p_{k}(0)=1, p_{i}=0, i=0,1,2, \ldots ., i \neq k$. It is necessary to take into account the standardised condition $\sum_{i=0}^{n} p_{i}(t)=1$.

At a constant rate of the flow of termination and generation and a finite number of states of the system $n+1$ there will be an idle mode. This claim is based on the fact that the set W of all states of the process of termination and generation is ergodic, since all of the system states and the subsets of states are transitive. Therefore the system S with a finite number of states $n+1$, in which a process of termination and generation of these flows is executed at constant rate, is the simplest ergodic system [3]. The graph of the system states is shown in Fig.3.


Fig. 3. Graph of the system states
The marginal probabilities of the states of the simplest ergodic process of termination and generation, found in the idle mode, can be obtained from the system of equations (7).

$$
\begin{align*}
& \left\{\begin{array}{l}
\frac{d p_{0}(t)}{d t}=\mu_{1}(t) \cdot p_{1}(t)-\lambda_{0}(t) \cdot p_{0}(t) \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
\frac{d p_{i}(t)}{d t}=\lambda_{i-1}(t) \cdot p_{i-1}(t)+\mu_{i+1} \cdot p_{i+1}(t)-\left(\lambda_{i}(t)+\mu_{i}(t)\right) \cdot p_{i}(t), \quad i=1,2, \ldots,(n-1)
\end{array}\right.  \tag{7}\\
& \frac{d p_{n}(t)}{d t}=\lambda_{n-1}(t) \cdot p_{n-1}(t)-\mu_{n}(t) \cdot p_{n}(t)
\end{align*}
$$

For the system (7) $p_{i}(0) \geq 0, i=0,1,2, \ldots, n$ and $\sum_{i=1}^{n} p_{i}(0)=1$. In the same system all rates are constant. Therefore the derivatives of the system states are 0. From the system (7) we obtain the results (8) and (9):

$$
\begin{align*}
& p_{i}=\frac{\lambda_{i-1}}{\mu_{i}} \cdot p_{i-1}=\frac{\lambda_{i-1}}{\mu_{i}} \cdot \frac{\lambda_{i-2}}{\mu_{i-1}} \ldots \frac{\lambda_{0}}{\mu_{1}} \cdot p_{0}=p_{0} \cdot \prod_{k=1}^{i} \frac{\lambda_{k-1}}{\mu_{k}}, \quad i=1,2, \ldots, n  \tag{8}\\
& \text { Then } \sum_{i=0}^{n} p_{i}=p_{0}+p_{0} \cdot \sum_{i=1}^{n} \prod_{k=1}^{i} \frac{\lambda_{k-1}}{\mu_{k}}=1, \quad \text { т.e. } p_{0}=\frac{1}{1+\sum_{i=1}^{n} \prod_{k=1}^{i} \frac{\lambda_{k-1}}{\mu_{k}}} .
\end{align*}
$$

We have to find the probabilities of the system states in idle mode, if the rate of new ambulances entering the car park is a constant, i.e. $\lambda(t)=\lambda=$ const.

If there are no more than $n$ ambulances in the car park, then from (9) we obtain the result [7]:
$p_{0}=\frac{1}{1+\sum_{i=1}^{n} \prod_{k=1}^{i} \frac{\lambda_{k-1}}{\mu_{k}}}=\frac{1}{1+\sum_{i=1}^{n} \frac{\alpha^{i}}{i!}}=\frac{1}{\sum_{i=0}^{n} \frac{\alpha^{i}}{i!}}, \quad$ where $\alpha=\frac{\lambda_{k-1}}{\mu_{k}}$.
Therefore $\quad p_{i}=p_{0} \cdot \prod_{k=1}^{i} \frac{\lambda}{\mu_{k}}=p_{0} \prod_{k=1}^{i} \frac{\lambda}{k \cdot \mu}=p_{0} \cdot \frac{\alpha^{i}}{i!}, \quad i=0,1,2, \ldots ., n$.

We find the mathematical expectation and dispersion of the random variable $X(t)$ (number of ambulances in operation) in idle mode by the formulae (6).

Table 1. Probabilities of the system states at $\mu=8.10^{-6} \mathrm{a} / \mathrm{h}$.

| $\lambda=10^{-6}$ | $\lambda=2.10^{-6}$ | $\lambda=4.10^{-6}$ | $\lambda=6.10^{-6}$ | $\lambda=7.10^{-6}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\alpha=0.125$ | $\alpha=0.25$ | $\alpha=0.5$ | $\alpha=0.75$ | $\alpha=0.875$ |
| $p_{8}=1.305 * 10^{-12}$ | $p_{8}=2.947 * 10^{-10}$ | $p_{8}=5.876 * 10^{-8}$ | $p_{8}=117 * 10^{-8}$ | $p_{8}=355 * 10^{-8}$ |

The computations are performed on MAPLE.

## CONCLUSIONS BASED ON THE RESULTS FROM TASK TWO

If $\alpha<1$, then the termination mode is present in the system and the probability that at some point in time $t \rightarrow \infty$ there are eight ambulances in operation at the car park at the EMCC is $p_{8} \approx 0$. If $\alpha>1$, then the generation mode is present in the system and by increasing the values of $\alpha$ the mathematical expectation takes values greater than 1 , indicating an increase in the number of ambulances in operation at the car park at the EMCC. If $\lambda=2.10^{-5}$ and $\mu=8.10^{-6}$, then $\alpha=2.5$ and the mathematical expectation of the number of ambulances will be 2.5 . Only when $\lambda=6.10^{-5}$ and $\mu=8.10^{-6}$ the mathematical expectation (the number of ambulances in operation) reaches the number 6, which will ensure timely service to the patients.

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