Modelling of the heat exchange and accumulation processes of a green roof

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Abstract: *A mathematical model describing the heat exchange and accumulation processes of a green roof has been developed. It takes into account the convective heat transfer through the grass surface and the roof of the building, the accumulated and reflected solar energy, the radiated energy as well as the energy loses from evapotranspiration. The model allows to determine the soil temperature of the soil layers, which allows to calculate the energy loses of the building through the roof construction.*
Keywords: *green roof, mathematical model, evapotranspiration*

INTRODUCTION

The use of green roofs as a means for reducing the energy loses becomes popular nowadays. They reduce significantly the heat exchange through the roof on the one hand and maintain low roof temperature during the summer on the other, which reduces the urban heat island effect [2]. In order to properly construct such construction with minimal investments, it is necessary to simulate and predict the heat exchange processes through it, depending on the local climate. If the depth of the soil layer is too thin, the green roof might not provide the required insulation parameters, and if it is too thick, this would increase significantly the initial construction investments, because of the great roof mass.

In [5] has been developed a mathematical model, describing the heat exchange through a green roof, but it does not account for the evapotranspiration from the grass surface and the sun radiation and they both play a major role in the heat exchange and accumulation processes of a green roof.

The goal of this study is to develop a mathematical model, which describes the heat exchange and accumulation processes of a green roof, which takes into consideration the evapotranspiration and the solar radiation.

JUSTIFICATION OF THE USED APPROXIMATIONS AND DEPENDENCIES

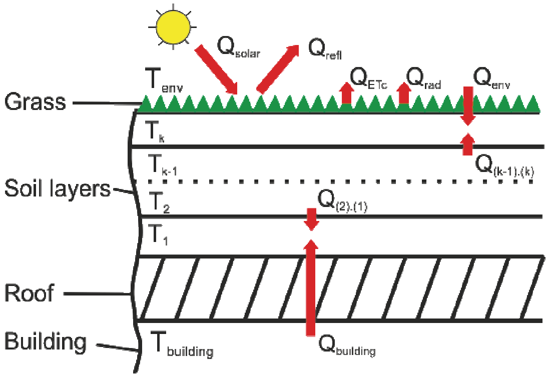

Figure. 1. Energy streams through the green roof

The generalized scheme of the energy streams through a green roof are shown in fig. 1. The building has a flat concrete roof, on which a soil layer covered with green grass has been placed. In order to model it, the soil has been divided into k sub-layers, having temperatures T_1, T_n,... T_k respectively. Since the heat transfer process through a green roof is non-stationary, the new layer temperature is calculated using the two energy streams, coming from the two sides of the layer:

$$T_i^{+t} = T_i + \Delta t \frac{Q_{(i-1).(i)} + Q_{(i+1).(i)}}{C_{soil}.m_{soil.layer}} F \text{ , } °C \text{ ,} \qquad (1)$$

where $Q_{(i-1).(i)}$ and $Q_{(i+1).(i)}$ are the energy streams coming from the ($i-1$)-th and the ($i+1$)-th layers, W;

T_i - the temperature of the i-th layer in the previous moment, $°C$;

Δt - the time interval since the previous moment, sec;
F - the heat exchange surface, m^2;
C_{soil} - the specific heat capacity of the soil, $J.kg^{-1}.K^{-1}$;
$m_{soil.layer}$ - the mass of one soil layer, kg.

The energy streams coming from the environment and from the building is calculated using relations (2), (3) and (4)] where the index of the soil sub-layer streams shows their direction: $(i+1).(i)$ means the direction is from the ($i+1$)-th to the i-th layer:

$$Q_{env} = Q_{conv.env} + Q_{solar} - Q_{ET_C} - Q_{rad} - Q_{refl}\ ,\ W.m^{-2}, \tag{2}$$

$$Q_{conv.env} = \frac{\left(T_{env} - T_k\right)}{\dfrac{\delta_{soil.layer}}{\lambda_{soil}} + \dfrac{1}{\alpha_{green.roof}}}\ ,\ W.m^{-2} \tag{3}$$

$$Q_{building} = \left(T_1 - T_{building}\right) / \left(\frac{1}{\alpha_{roof}} + \frac{\delta_{roof}}{\lambda_{roof}}\right),\ W.m^{-2}, \tag{4}$$

$$Q_{(i+1).(i)} = \frac{T_{i+1} - T_i}{R_i}\ ,\ W.m^{-2} \tag{5}$$

where T_{env} is the temperature of the environment, $°C$;
$T_{building}$ - the temperature inside the building, $°C$;
$Q_{conv.env}$ - the convective heat transfer energy flux from the environment to the topmost soil layer, $W.m^{-2}$;
Q_{solar} - the incoming solar energy flux, $W.m^{-2}$;
Q_{ET_C} - the energy flux used for evapotranspiration, $W.m^{-2}$;
Q_{rad} - the radiated energy flux, $W.m^{-2}$;
Q_{refl} - the reflected solar energy flux, $W.m^{-2}$;
α_{roof} - the heat transfer coefficient of the roof from the bottom side, $W.m^{-2}.K^{-1}$;
δ_{roof} - the width of the roof, m;
λ_{roof} - the heat conductivity of the roof, $W.m^{-1}.K^{-1}$;
R_i - the heat resistance of the i-th layer, $m^2.K.W^{-1}$.

The energy Q_{ET_C} is calculated based on the evapotranspirated water ET_C from the grass surface, using the Penman-Monteith method to determine the reference crop evapotranspiration ET_0 [1]:

$$Q_{ET_C} = K_C.ET_0.E_{evp}\ ,W.m^{-2}; \tag{6}$$

$$ET_0 = \frac{0.408.\Delta.\left(R_n - G\right) + \gamma \dfrac{900}{T_m + 273.15}.s.\left(P_s - P_a\right)}{\Delta + \gamma.\left(1 + 0.34.s\right)}\ ,\ mm/day\ , \tag{7}$$

where ET_0 the reference crop evapotranspiration, mm/day;
R_n - the net radiation flux, $MJ.m^{-2}.day^{-1}$;
G - the sensible heat flux into the soil, $MJ.m^{-2}.day^{-1}$;
T_m - the mean daily temperature, $°C$;
s - the mean daily wind velocity, $m.s^{-1}$;
P_s - the saturated vapor pressure, kPa;
P_a - the partial pressure of the air, kPa;

Δ - the slope of the saturated vapor pressure curve, $kPa.°C^{-1}$;
γ - the psychometric constant, $kPa.°C^{-1}$;
K_C - the crop coefficient, depending on the grass type;
E_{evp} - the energy required to evaporate the evapotranspirated water, J;

In order to calculate R_n and Q_{solar} it is necessary to determine the sun orientation angle θ_i to the green roof surface for each hour of the day according to [5]:

$$\cos\theta_i = \sin\delta.\sin\phi.\cos\beta + \sin\delta.\cos\phi.\sin\beta.\cos A_{ZS} + \cos\delta.\cos\phi.\cos\beta.\cos\omega - \\ -\cos\delta.\sin\phi.\sin\beta.\cos A_{ZS}.\cos\omega - \cos\delta.\sin\beta.\sin A_{ZS}.\sin\omega; \quad (8)$$

$$\delta = 23.45\frac{\pi}{180}\sin\left[2\pi.\left(\frac{284+n}{356.25}\right)\right], \text{deg} \quad (9)$$

where δ the declination angle, deg;
ω - the hour angle, deg;
ϕ - the latitude angle, deg;
β - the slope angle of the surface and the horizontal, deg;
A_{ZS} - the azimuth angle (rotation from north), deg;
n - the day of the year.

In order to determine the whole solar radiation flux, falling on the green surface, are used the following dependencies [3]:

$$AM = 1/\cos(90-\delta) + 0.50572.(96.07995-(90-\delta))^{-1.6364},\ m, \quad (10)$$

$$I_G = 1{,}1.1353.0{,}7^{(AM^{0{,}678})},\ W.m^{-2}, \quad (11)$$

$$I_{G.angle} = I_G.\sin(\theta_i),\ W.m^{-2} \quad (12)$$

$$Q_{solar} = I_{G.angle}.(1-0.49.Cl/10),\ W.m^{-2} \quad (13)$$

where AM is the airmass of the atmosphere, m,
I_G - the global radiation falling on a perpendicular surface, $W.m^{-2}$;
$I_{G.angle}$ - the global radiation falling on a surface with angle θ_i, $W.m^{-2}$;
Cl - the sky cloudiness taking values between 0 and 10.

The radiated energy flux from the grass surface could be determined with:

$$Q_{rad} = \varepsilon.\sigma.(T_k^4 - T_{env}^4),\ W.m^{-2},$$

where ε is the thermal emissivity of the grass surface (0.97 - 0.98);
σ - the Stephan-Boltzmann constant, $W.m^{-2}.K^{-4}$.

The reflected energy flux from a grass surface is about 26% of the solar one, which makes $Q_{refl} = 0.26.\mathrm{Q_{solar}}$.

ALGORITHM FOR MODELING OF THE HEAT EXCHANGE AND ACCUMULATION PROCESSES OF A GREEN ROOF

The finite elements method is used in the developed model [4,6]. For this purpose, the time and space are divided into very small section/interval and the energy exchange is described for each sections and each moment of time.

The main algorithm of the model is presented in figure 2. The initial temperatures of the soil layers are initialized in block 1. The declination angle δ and the surface angle θ_i are calculated in block 2, according to dependencies (8) and (9). The global solar radiation falling on the surface is calculated in block 3, according to dependencies (10) ÷ (13). The energy flux from evapotranspiration is determined in block 4 using dependencies (6) and (7). The reflected and radiated energy fluxes are calculated in block 5 and the convective heat transfer flux - in block 6. The total environment energy entering the topmost soil layer is determined in block 7. The subprogram for actualization of the temperatures of all the soil layers is called in block 8. Block 2-9 are repeated until the maximal time moment is reached.

The subprogram for actualization of the temperatures of the soil layers is shown in figure 3. The sensible heat flux from the second to the first layers is determined in block 2. The temperature of the first layer after time Δt is actualized in block 3. The temperatures of the rest of the soil layers is calculated in cycle in blocks 5-7. The sensible heat flux from the $(k-1)$-th to the k-th layer is calcualted in block 8, and the temeprature of the topmost layer - in block 9.

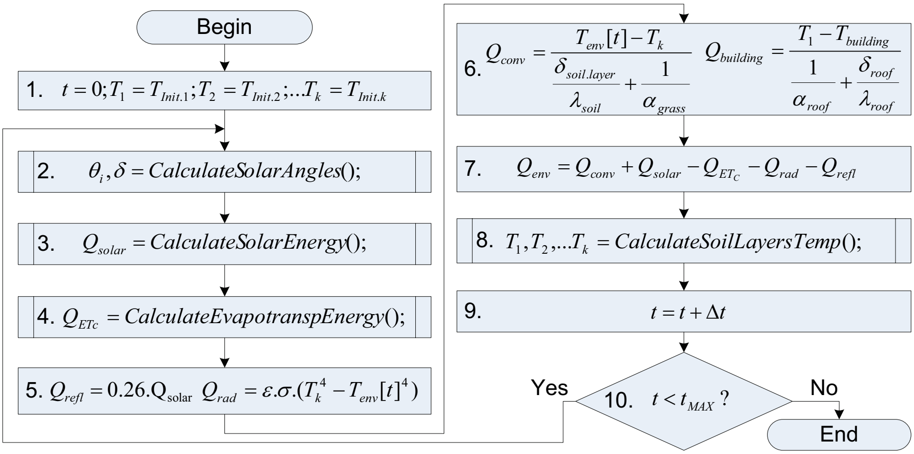

Figure. 2. Main algorithm of the model for heat exchange and accumulation processes of a green roof

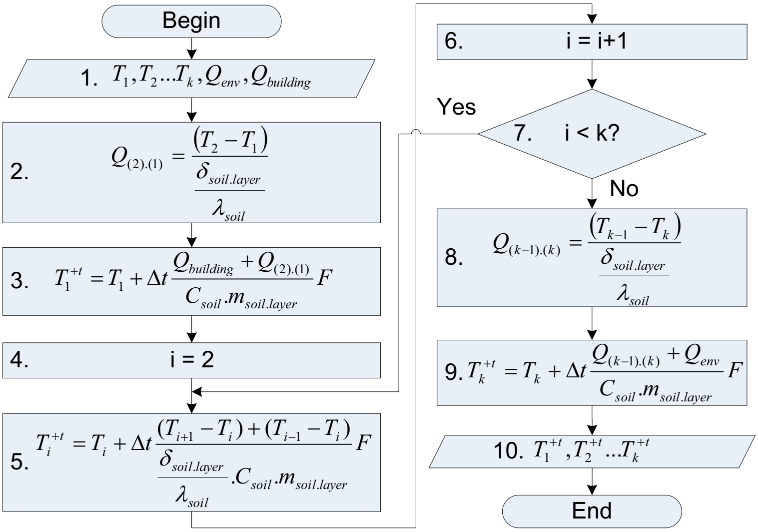

Figure. 3. Subprogram for actualization the temperatures of the soil layers for a certain moment of time

CONCLUSIONS AND DISCUSSION

A mathematical model, describing the heat exchange and accumulation processes of a green roof has been developed in this study. The model takes into account the influence of the solar radiation, the evapotranspiration, the radiation from the grass surface, thus considering all the factors, which influence the investigated process.

The model also considers the declination angle of the sun and the surface angle, which allows to determine the solar radiation and evapotranspiration for the whole year, for any climate conditions, as long as the required climate data is available: environment temperature and humidity, sky cloudiness and wind velocity.

The calculated evapotranspiration from a green roof is accurate as long as there is enough moisture in the soil. The model could be used if that condition is not met as well, but the crop coefficient K_C and the soil thermal characteristics need to be updated to consider the water stress.

Using the developed model, energy loses from a green roof could be predicted, depending on the depth and the type of the soil.

It is necessary to conduct experimental studies in order to verify the model's accuracy and this is an objective for future investigations.

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Докладът е рецензиран.