Investigation of the precision of a mathematical model describing the heat exchange and accumulation processes of a green roof

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Abstract: *An investigation of the efficiency of the model has been carried out on the basis of a developed model of the heat exchange and accumulation processes of a green roof and the results of an experimental study. A criterion for determination of the optimal daily crop coefficient has been synthesized, which considers the water stress. The mean daily crop coefficients have been estimated for the duration of the investigation. Using them, a simulation of the developed model has been carried out. The results have been processed statistically which proves the efficiency of the suggested mode.*

Keywords*: green roof, mathematical model, evapotranspiration, heat exchange.*

INTRODUCTION

The reduction of the energy loses of the dwelling buildings is a priority of the contemporary society. The additional insulation of the buildings' walls and roofs is a common occurrence, but it requires additional investments. Another alternative is to use green roofs, who's cost price does not increase significantly the initial investment of the newly constructed buildings. It allows to significantly reduce the roof temperature during the summer, which on the one hand reduces the energy loses and on the other - reduces the urban heat island effect [4].

The goal of this study is to investigate the efficiency of an already developed model of the heat exchange and accumulation processes of a green roof by comparing its results with those received in an experimental study. A statistical processing also needs to be conducted in order to determine if the modeled results are close to the experimental ones.

OBJECT OF THE INVESTIGATION

The object of the investigation is a green roof construction in the village of Bojichen, Ruse district, on which has been installed a monitoring system. It is used to collect information about the temperature variation soil in depth. The depth of the soil layer is 12 cm and it's type - Chernozem. The system monitors the temperature of the soil at depths 3 cm and 10 cm, as well as the environment and the building temperatures. The results of the investigation has been presented in [3].

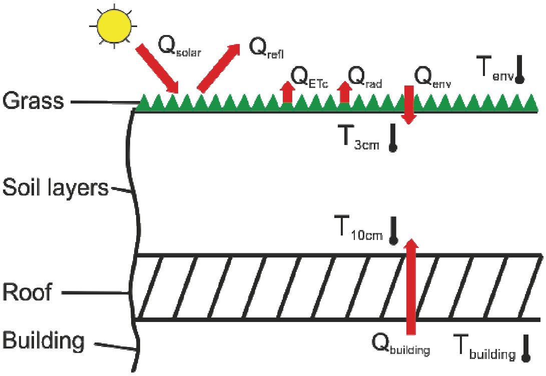

Figure 1. General scheme of the heat exchange and accumulation processes in a green roof.

A mathematical model has been developed, which describes the heat exchange and accumulation processes of a green roof, which has been presented in [2]. The model has been used to simulate the object processes for the same period of time and compare the experimental and the simulated soil temperatures.

In order to determine the evapotranspiration from the grass surface with the developed model, the following equation is used [2]:

$$Q_{ET_C} = K_C.ET_0.E_{evp}\ ,\ W.m^{-2} \qquad (1)$$

where ET_0 is the reference crop evapotranspiration, mm/day;

K_C - the crop coefficient, which depends on the crop type and it's height;

E_{evp} - the energy, required to evaporate the evapotranspirated water, J;

The evapotranspiration Q_{ET_C} is calculated properly only if the water concentration of the soil is adequate. This condition has not been met for most part of the experiment. That's why it is necessary to modify the crop coefficient K_C, so that it considers the water stress.

SELECTION OF A CRITERION FOR ESTIMATION OF THE CROP COEFFICIENT

In order to estimate the mean daily crop coefficients K_C, considering the mean daily water concentration, it is necessary to select an appropriate criterion. There are a number of possibilities like for example a minimal sum of squares or closes daily minimal and maximal values of the temperature.

Since this model is aimed to calculate the energy loses through the roof construction, the following criterion is suggested:

$$Crit = \sum_{0}^{k} (Q_{building.Mod} - Q_{building.Exp})^2 = \min , \qquad (2)$$

where $Q_{building.Mod}$ is the heat flux coming from the building towards the soil layer, calculated using the data from the model, $W.m^{-2}$;

$Q_{building.Exp}$ - the heat flux coming from the building towards the soil layer, calculated using the experimental data, $W.m^{-2}$;

k - the seconds count for the current day, sec.

The energy flux $Q_{building}$, coming from the building, is determined according to the Fourier's law, and for $T_{soil.10cm}$ are used the values from the model and from the experiment:

$$Q_{building} = \left(T_{building} - T_{soil.10cm}\right) / \left(\frac{1}{\alpha_{roof}} + \frac{\delta_{roof}}{\lambda_{roof}} + \frac{\delta_{soil}}{\lambda_{soil}} \right), \; W.m^{-2}, \qquad (3)$$

where $T_{building}$ is the temperature inside the building, $°C$;

$T_{soil.10cm}$ the soil temperature at 10 cm depth, $°C$;

α_{roof} the specific heat transfer coefficient of the building ceiling, $W.m^{-2}.K^{-1}$;

δ_{roof} the width of the roof slab, m;

λ_{roof} the heat conductivity coefficient of the roof slab, $W.m^{-1}.K^{-1}$;

δ_{soil} the distance between the roof slab and the soil layer at depth 10 cm, m;

λ_{soil} the heat conductivity of the soil, $W.m^{-1}.K^{-1}$.

METHOD FOR INVESTIGATION

In order to estimate the mean daily crop coefficients K_C, using the criterion from equation (2), the following method has been applied: a simulation of the developed model is conducted for each of the investigated days, using a crop coefficient K_C in the interval (0.5 - 1.0) with incremental step 0.05. The optimal coefficient is determined by finding the minimal value of the criterion $Crit$. After finding the optimal K_C, this step is repeated for the rest of the days.

After all of the mean daily crop coefficients K_C are determined, a new simulation of the model for heat exchange and accumulation processes of a green roof for the whole time period should be carried out, using the determined values for K_C accordingly. Then the variation of the temperature of the soil at depth 10 cm, and the building energy loses could be taken down for further processing.

RESULTS

A software application has been developed in the Visual Studio 2010 environment, implementing the developed model. The application looks for optimal values of the crop coefficient K_C, according to the suggested criterion (equation (2)) and method.

In order to determine the energy loses from evapotranspiration, meteorological data for environment temperature, humidity, sky cloudiness and air velocity in the city of Ruse have been used, for the time of the investigation [6]. The heat characteristics of the soil, used in the modeling, are formed based on the data in table 1, by using a base water concentration of 15 %.

Table 1.

Heat characteristics of Chernozem soil and water [5].

Attribute	Value
Density of dry Chernozem	1290 $kg.m^{-3}$
Specific heat capacity of dry Chernozem	1041 $J.kg^{-1}.K^{-1}$
Heat conductivity of dry Chernozem	0.31 $W.m^{-1}K^{-1}$
Density of water	995 $kg.m^{-3}$
Specific heat capacity of water	4185 $J.kg^{-1}.K^{-1}$
Heat conductivity of water	0.58 $W.m^{-1}K^{-1}$

The determined optimal mean daily values of the crop coefficient K_C, according to the suggested method, and the value of the criterion for optimality are shown in table 2. Using these values the temperature curve of the soil at depth 10 cm has been built, which can be seen in figure 2.

Table 2.

Values of the mean daily crop coefficient K_C and the corresponding criterion for optimality.

Day №	1	2	3	4	5	6	7	8	9	10
K_C	0.75	0.7	0.85	0.9	0.75	0.95	1.0	0.9	0.9	0.7
$Cirt$	280	534	460	416	166	73	347	271	342	196

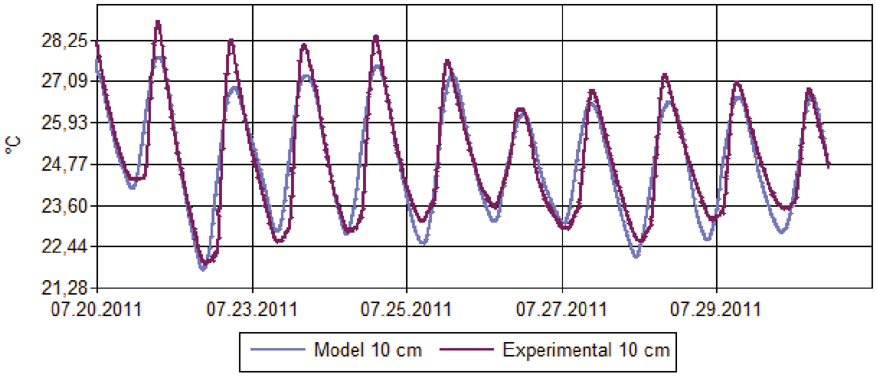

Figure 2. Comparison between the experimental and the model simulated values of the soil temperature at depth 10 cm

In table 2, it could be seen that the crop coefficient K_C varies in the interval (0.7 ÷ 1.0), as a result of the water stress. These results are expected, considering the grass surface hasn't been irrigated regularly, which leads to a non-constant crop coefficient.

In normal exploitation of the green roof and regular irrigation, the crop coefficient should be in very close bounds and should depend mainly on the crop's type and height.

From the shown figure it could be seen that the model describes the heat exchange and accumulation processes of a green roof relatively accurate. At some points there are larger deviation which could be caused by the face that the meteorological data is for the city of Ruse, which is 17 km away from the investigated object. Another factor is that when the water concentration is reduced, this also influences the soil's thermal characteristics.

Using equation (3) the heat flow through the ceiling of the building has been calculated, which is shown in figure 3. The total energy loses per square meter for the whole period are 9867 $W.m^{-2}$, according to the experimental data, and 8922 $W.m^{-2}$ - according to the model, which is about 0.0027 $kW.h$ and 0.0025 $kW.h$ energy per square meter respectively.

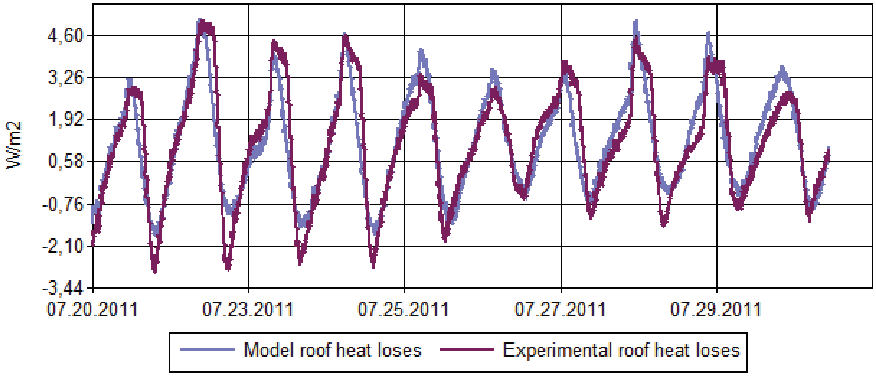

Figure 3. The heat flow through the ceiling of the building, according to the experimental and the model simulated data

Using the heat flow data, a statistical t-Test for equality of the mean values of the two independent groups have been made: H0: $\overline{Q_{building.\exp}} = \overline{Q_{building.\mathrm{mod}}}$. The results of the test are shown in table 3.

Table 3.
Results from the t-Test.

Attribute	**Value**
$\overline{Q_{building.\exp}}$	1,157749
$\overline{Q_{building.\mathrm{mod}}}$	1,187694
r	0,909894
s_{Exp}	1,799359
s_{Mod}	1,611817
s_{Total}	1,708164
k	11254
n_{Total}	53,04715
F	1,246247
t	0,929963
$t_{table}(k \Rightarrow \infty)$	1,96

Considering that the t-Test value is $t = 0{,}929963$, which is less than the corresponding criterion value ($t_{table}(k \Rightarrow \infty) = 1{,}96$) for $\alpha = 0.05$, the Null hypothesis does not conflict with the experimental data. The correlation coefficient $r = 0{,}909894$ is larger than 0 and close to 1, so it could be considered that the mathematical model for heat exchange and accumulation describes the investigated processes precisely.

CONCLUSION

A criterion for optimality has been developed, which is used to estimate the optimal mean daily value of the crop coefficient K_C by taking into account the water stress during evapotranspiration. A method has been suggested for determination of the optimal values of K_C when the criterion has a minimal value. By modeling the developed in [2] model, describing the heat exchange and accumulation processes of a green roof, a simulation has been carried out, in order to determine the mean daily crop coefficients.

By using the calculated K_C values the whole period of investigation has been simulated and the soil temperature curve of the green roof has been taken down. It has been compared graphically and statistically with the experimental soil temperature curve at 10 cm depth. The statistical processing shows that the model describes the heat exchange and accumulation processes in a green roof accurately.

The model could be used for design of green roofs as well as for control of the microclimate of a building, as long as the required climatic data is available.

Additional studies are required in order to test the model in winter conditions, which is an object for future investigations.

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Докладът е рецензиран.