# Modeling of indoor air composition time variation during the night in naturally ventilated occupied spaces

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***”Modeling of indoor air composition time variation during the night in naturally ventilated occupied spaces”:*** *presents a closed non-steady state one dimensional mathematical model for evaluation of indoor air composition time variation in occupied spaces with non-organized non-controllable natural ventilation. It presents as well an analysis of the mathematical model by which are identified both the factors and the level of there impact on the variation of system mass and composition*

***Key words: natural ventilation, metabolic $CO_2$ time variation***

***”Моделиране на изменението във времето на състава на въздуха през нощта в обитаеми помещения с естествена вентилация”:*** *представя затворен нестационарен едномерен математически модел за определяне на състава на въздуха (газовата смес) в обитаеми помещения с неорганизирана неконтролируема естествена вентилация. Тук е пресдтавен, също така, и анализ на математическия модел, чрез който са идентифицирани факторите и степента на тяхното влияние върху изменението на масата и на състава на газовата смес в помещението.*

***Ключови думи: естествена вентилация, изменение на метаболния $CO_2$ във времето.***

## INTRODUCTION

Tracer gas technique with metabolic $CO_2$ as a tracer gas is used widely over the last decades for ventilation measurements in occupied spaces [3, 4, 5, 7, 8, 9] even with natural ventilation.

Typically its application is based on the $CO_2$ mass balance equation in volume units

$$\frac{dX_r}{dt} = \frac{10^3 \dot{G}}{V_r} + X_a \frac{\dot{V}_a}{V_r} - X_r \frac{\dot{V}_r}{V_r}, \tag{1}$$

where $X_r$ and $X_a$ are $CO_2$ volume fractions in the room air and outdoor air (ppm), respectively, $V_r$ is room air volume ($m^3$), $\dot{G}$ is $CO_2$ generation rate (*l*/s) at the room conditions, $\dot{V}_r$ is the volume flow rate at which the indoor air escapes from the room ($m^3$/s), and $\dot{V}_a$ is the volume flow rate at which the outdoor air enters the room ($m^3$/s).

System mass balance equation is taken implicitly in the form

$$\dot{V}_r = \dot{V}_a, \tag{2}$$

In the ASTM E 741-06 standard [1] the tracer gas technique is described with details and the limitations of its applicability are presented and explained. The ASTM D 6245–07 standard [2] presents the conditions under which metabolic $CO_2$ could be used for ventilation measurements following the ASTM E 741-06 tracer gas technique. There is stated that only the decay technique can be used for ventilation measurements in a single zone, i.e the system exchanges air only with outdoors. Unfortunately the metabolic $CO_2$ tracer gas technique, based on equations 1 and 2, is applied for ventilation measurements in occupied spaces with natural ventilation even for build-up intervals without analyzing the potential uncertainties, which could affect the result significantly since in naturally ventilated occupied spaces very often $ACH = \frac{\dot{V}_r}{V_r} \le 0.5$.

## MATEMATICAL MODEL

Schematic diagram of the system and the surroundings is presented on Figure 1. The indoor air (r) is the system under investigation. The system exchanges mass with the surroundings, i.e. some amount of indoor air escapes to outdoor ($\dot{m}_{r,a}$) and/or to the neighboring spaces ($\dot{m}_{r,d}$) and simultaneously some amount of outdoor air ($\dot{m}_a$) and/or air from the neighboring spaces ($\dot{m}_d$) enters into the system. The system is in a ceaseless interaction as well with the occupants who consume oxygen from it and release back to it heat, mass, carbon dioxide gas and water vapor. During this interaction system composition ($X_{j,r}$) is changed. The outdoor parameters absolute pressure ($P_a$), absolute temperature ($T_a$) and composition vector ($X_{j,a}$) varies with time, during the day and annually, due to weather changes and they cause variation of corresponding system parameters ($P_r$, $T_r$, $X_{j,r}$) and air parameters in the neighboring spaces ($P_d$, $T_d$, $X_{j,d}$). As a result of those interactions all system parameters, including system mass $m_r$, vary with time to different extent.

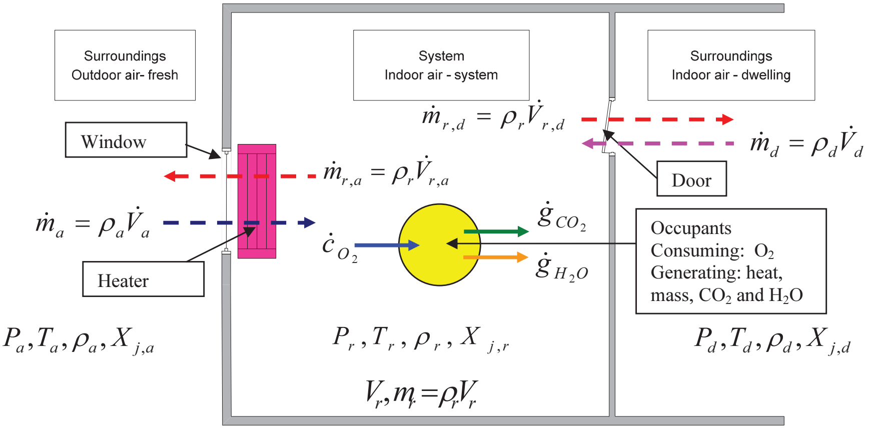

*Figure 1: Schematic diagram of the system and its interaction with the surroundings*

For establishing of a procedure for analysis of indoor air composition time variation one needs several models and conservation equations, i.e. model of system properties, model of occupants' metabolic process - for the required energy by the body from one side and for the rate of overall reactions taking place within the body from other, i.e. for $O_2$ consumption rate ($\dot{c}_{O2}$), and for the generation rates of $CO_2$ ($\dot{g}_{CO2}$) and $H_2O$ ($\dot{g}_{H2O}$), and conservation equations for system mass ($m_r$) and for the mass of each species ($X_{j,r}$).

Models for both system properties and occupants' metabolic process are already prepared and presented elsewhere [6]. Here will be presented conservation equation for system mass and mass balance equation for $CO_2$. Mass balance equations of $O_2$ and $H_2O$ are similar to $CO_2$ mass balance equation and will not be presented here.

System mass balance equation in mass units, assuming that there is only one neighboring space in the dwelling, reads:

$$\frac{dm_r}{dt} = \dot{m}_M + \dot{m}_a + \dot{m}_d - \dot{m}_r\,, \tag{3}$$

where $\dot{m}_M = \dot{g}_{CO2} + \dot{g}_{H2O} - \dot{c}_{O2} = \rho_M \dot{V}_{M,r}$ is the source of mass in the system due to occupants' metabolism (kg/s), $\dot{V}_{M,r}$ is the volume flow rate of mass source at room conditions ($m^3$/s), and $\dot{m}_r = \dot{m}_{r,a} + \dot{m}_{r,d} = \rho_r \dot{V}_r$ is the total amount of air leaving the system.

Taking into account that system volume is constant and evaluating system density by ideal gas equation of state equation 3 can be written in volume units as follows

$$\frac{d}{dt}\left(\frac{P_r}{R_rT_r}\right)=\frac{P_r}{R_rT_r}\frac{\dot{V}_{M,r}}{V_r}+\frac{P_a}{R_aT_a}\frac{\dot{V}_a}{V_r}+\frac{P_d}{R_dT_d}\frac{\dot{V}_d}{V_r}-\frac{P_r}{R_rT_r}\frac{\dot{V}_r}{V_r} \tag{4}$$

After expanding the LHS of equation 4 and solving it against the ventilation rate $\frac{\dot{V}_r}{V_r}$ one gets

$$\frac{\dot{V}_r}{V_r}=\left(\frac{P_a}{P_r}\frac{R_r}{R_a}\frac{T_r}{T_a}\right)\frac{\dot{V}_a}{V_r}+\left(\frac{P_d}{P_r}\frac{R_r}{R_d}\frac{T_r}{T_d}\right)\frac{\dot{V}_d}{V_r}+\frac{\dot{V}_{M,r}}{V_r}-\frac{\dot{V}_{r,dP}}{V_r}+\frac{\dot{V}_{r,dT}}{V_r}+\frac{\dot{V}_{r,dR}}{V_r}, \tag{5}$$

where $\frac{\dot{V}_{r,dP}}{V_r}=\frac{1}{P_r}\frac{dP_r}{dt}$, $\frac{\dot{V}_{r,dT}}{V_r}=\frac{1}{T_r}\frac{dT_r}{dt}$, and $\frac{\dot{V}_{r,dR}}{V_r}=\frac{1}{R_r}\frac{dR_r}{dt}$.

Equation 5 has a form which allows for identification of all factors which have impact on the system mass balance. There are four groups of factors influencing system mass balance:

- F1: the relative time variation of system pressure ($\dot{V}_{r,dP}$), system temperature ($\dot{V}_{r,dT}$) and system composition ($\dot{V}_{r,dR}$);
- F2: the difference between the system parameters and their corresponding parameters of the outdoor air and the air in the neighboring space, i.e. the ratios $\frac{P_a}{P_r},\frac{T_r}{T_a},\frac{R_r}{R_a}$ and $\frac{P_d}{P_r},\frac{T_r}{T_d},\frac{R_r}{R_d}$;
- F3: the mass source strength, i.e. $\frac{\dot{V}_{M,r}}{V_r}$;
- F4: the amount of air entering the system from the neighboring space $\frac{\dot{V}_d}{V_r}$

Based on the assumptions for system homogeneity and perfect mixing $CO_2$ mass balance equation in mass units has the same form as the system mass conservation equation. In terms of mass fraction it has the form

$$\frac{d(\rho_rV_rY_r)}{dt}=\dot{g}_{CO_2}+\rho_a\dot{V}_aY_a+\rho_d\dot{V}_dY_d-\rho_r\dot{V}_rY_r, \tag{6}$$

In terms of volume fraction (ppm) $CO_2$ mass balance equation reads

$$\frac{d}{dt}\left(\frac{P_r}{T_r}X_r\right)=\frac{P_r}{T_r}\frac{10^3\dot{G}_{CO_2}}{V_r}+\frac{P_a}{T_a}X_a\frac{\dot{V}_a}{V_r}+\frac{P_d}{T_d}X_d\frac{\dot{V}_d}{V_r}-\frac{P_r}{T_r}X_{CO_2,r}\frac{\dot{V}_r}{V_r}, \tag{7}$$

where $\dot{G}_{CO_2}=\dot{g}_{CO_2}R_{CO_2}\frac{T_r}{P_r}$ is $CO_2$ generation rate in volume units at room conditions.

Here $\dot{g}_{CO_2}$ is expressed in (g/s) and $\dot{G}_{CO_2}$ is obtained in (*l*/s).

In equation 6 $Y_r$, $Y_a$, and $Y_d$ denote $CO_2$ mass fraction in the system, the outdoor air and in the air of the neighboring space. In equation 7 $X_r$, $X_a$, and $X_d$ denote $CO_2$ volume fractions in the same zones.

By expanding the LHS of equation 7 and rearrangement of the terms one gets

$$\frac{dX_r}{dt}=\frac{10^3\dot{G}_{CO_2}}{V_r}+\frac{T_rP_a}{T_aP_r}X_a\frac{\dot{V}_a}{V_r}+\frac{T_rP_d}{T_dP_r}X_d\frac{\dot{V}_d}{V_r}-X_r\left(\frac{\dot{V}_r}{V_r}+\frac{\dot{V}_{r,dP}}{V_r}-\frac{\dot{V}_{r,dT}}{V_r}\right) \tag{8}$$

From equation 8 it follows that the same group of factors (F1-F4) have impact on $CO_2$ mass balance equation. However the system air composition time variation has no explicit impact on $CO_2$ mass balance equation. In group F3 instead of mass source is present $CO_2$ generation rate. In addition to these differences a new factor appears in equation 8, F5:

$CO_2$ concentration in the surroundings, both outdoors and in the neighboring space.

It is clear that in order to use for ventilation measurements the set of equations 1 and 2 instead of 5 and 8 one must neglect several factors and assure several conditions.

## ANALYSIS OF THE IMPACT OF INFLUENCING FACTORS

In the nowadays buildings heating systems are equipped with thermostatic controls so indoor air temperature could vary with about 0.5 $^oC$ within half an hour.

The impact of system temperature relative variation in terms of ACH could be evaluated by the expression $ACH_{dT} = 3600\frac{\dot{V}_{r,dT}}{V_r} = \frac{3600}{T_r}\frac{dT_r}{dt}$. The above described small change of system temperature leads to $ACH_{dT} = 0.00339\ h^{-1}$. If this is omitted in equation 5 at ACH=0.1 the error is $\varepsilon_{dT} = \frac{ACH_{dT}}{ACH} = 3.388\%$. At ACH=0.5 the error is $\varepsilon_{dT}$=0.68%.

Under normal weather conditions atmospheric pressure varies with less then 50 Pa per hour. At atmospheric pressure of 90000 Pa $ACH_{dP} = \frac{3600}{P_r}\frac{dP_r}{dt} = 0.00056\ h^{-1}$. In case of omitting of this term in equation 5 at ACH=0.1 the error is $\varepsilon_{dP} = 0.556\%$ and at ACH=0.5 the error is $\varepsilon_{dP}$=0.111%.

Evaluation of the impact of the indoor air time variation will be done following the example presented in [6] where 4 people sleep in a bedroom with air volume of 39.48 $m^3$ for a period of 9 hours. In this situation, $X_r$ changes from 400 ppm to 9957 ppm, indoor air ideal gas constant changes from $R_r$=287.8553 J/(kg.K) to $R_r$=287.8147 J/(kg.K), so $ACH_{dR} = \frac{3600}{R_r}\frac{dR_r}{dt} = -0.0000156\ h^{-1}$. In case of omitting of this term in equation 5 at ACH=0.1 the error is $\varepsilon_{dR} = -0.016\%$ and at ACH=0.5 the error is $\varepsilon_{dR} = -0.003\%$.

At room temperature of 22 $^oC$ the ratio $\frac{T_r}{T_i}$ can be assumed equal to unity with an error of less then 0.7% when $|T_i - T_r| \le 2\ K$. In all other situations this ratio can not be assumed equal to unity in both equation 5 and equation 8.

When system pressure ($P_r$) differs from atmospheric pressure ($P_a$) with ±100 Pa, which is very rare, the ratio $\frac{P_a}{P_r} = 1.0 \pm 0.001111$, so this ratio can be always assumed equal to unity in both equation 5 and equation 8, since typically $|P_r - P_a| \le 10\ Pa$.

Ideal gas constant of outdoor air at $P_a$=94000 Pa, $t_a$=-15 $^oC$, relative humidity of 90% and $CO_2$ volume fraction of 390 ppm is $R_a = 287.2422\ J/(kg.K)$. Following the example presented in [6] at the beginning of the period $\left(\frac{R_r}{R_a}\right)_0 = 1.00213$ and at the end of the period $\left(\frac{R_r}{R_a}\right)_t = 1.00199$. This ratio can be always assumed equal to unity since with the increase of outdoor temperature it approaches unity.

In the example presented in [6] mass generation in the system is 0.01022 g/s and system mass is 43.979 kg. Under this conditions mass source strength is $ACH_M = 3600\frac{\dot{m}_M}{m_r} = 0.00084\ h^{-1}$. Omitting of this term in equation 5 at ACH=0.1 leads to an error of $\varepsilon_M = \frac{ACH_M}{ACH} = 0.83\%$. At ACH=0.5 the error is $\varepsilon_M = 0.17\%$.

The impact of $CO_2$ source strength on equation 8 will be evaluated by the following expression $\dot{V}_k = \frac{T_k}{X_k}\frac{10^3\dot{G}}{T_r}$, which gives the value of the volume flow rate of air entering the system from the surroundings (zone k), at the conditions in the zone $X_k$ and $T_k$, which produces the same impact on equation 8 as the $CO_2$ source term. Based on the example presented in [6] all occupants of the room generate 0.01984 g/s of $CO_2$, which at the conditions in the room is equivalent to 0.01177 l/s. This source has the power to change $CO_2$ volume fraction in the system with 17.89 ppm/min. At a temperature of the outdoor air of -15 $^{o}$C and $CO_2$ concentration of 400 ppm the flow rate of the outdoor air which will have the same impact on equation 8 as the $CO_2$ source term, expressed in ACH, is 2.35 $h^{-1}$. If outdoor air temperature is 15 $^{o}$C and $CO_2$ concentration is the same the flow rate of the outdoor air which will have the same impact on equation 8 as the $CO_2$ source term, expressed in terms of ACH, is 2.62 $h^{-1}$.

## CONCLUSIONS

System temperature variation has the strongest impact on system mass balance among the F1 group of factors. Time variation of both system composition and system pressure can be neglected in almost all practical situations.

Among the F2 group of factors temperature ratios in both equation 5 and equation 8 can be assumed equal to unity in very narrow interval. In all practical situations of naturally ventilated occupied spaces this ratio must be taken into account. In all practical situations both pressure ratios and ideal gas constant ratios can be assumed equal to unity.

Mass source strength (F3 factor) can be neglected in some situations, for example when ACH is closer to 0.5 or higher, or when in a huge room are sleeping one or two occupants.

CO2 source term has the strongest impact on CO2 mass balance equation and it must be evaluated as precisely as possible, by using the model presented in [6], and no limitations must be exerted on it when ACH is evaluated by build-up tracer gas technique applied to CO2 time variation records.

The impact of F4 factor can not be neglected a priory. Measures must be taken for identifying of its presence in each practical situation by measuring of Xd, Td, and (Pr-Pd).

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**The paper is reviewed.**