Solving Maximal Covering Location Problem (MCLP) by Using the Particle Swarm Optimization (PSO) Method

Darko Drakulić, Miroslav Marić, Aleksandar Takači

Abstract: *The Maximal covering location problem (MCLP) represents a very popular and important optimization problem. The MCLP is NP-hard problem and there are many heuristics for solving it, like Tabu search, Genetic algorithm, Lagrangian relaxation, etc.. This paper describes a new approach to solving MCLP by using a Particle Swarm Optimization (PSO) method. At the end, the paper presents the results of computational tests of this approach on several public instances of MCPL.*
Key words: *Optimization, PSO, MCLP*

INTRODUCTION

Location problems are very important class of optimization problems and solutions of them concerning optimal placement of facilities in order to maximize or minimize travel time, distance, transportation costs or some other parameter. Task of these problems is to find appropriate locations for facilities in the given space. Location problems are currently very popular because they have a large application, both in computer science and in the problems from real life.

Special subclass of location problems are covering location problems (CLP), where it is necessary to find a set of locations such that given function of distance is maximal or minimal. Site selection for facilities such as shops, gas stations, hospitals, or power plants are instances of CLP. In these cases, each building represents a given location and the problem is to find optimal location for *n* objects (shops, hospitals, etc.) such as they cover as much as possible buildings in given radius *r*. This paper describes a problem of maximization of the number of covered locations - maximal covering location problem (MCLP), but the problem could be to minimize the number of covered location (for example in case of garbage dumps, thermal power stations and other pollutants).

The MCLP is NP-hard problem [8], so it is not possible to find a solution in reasonable time for large dimension of the problem. Heuristic is a method for finding an approximate solution when classic methods fail to find any exact solution, as in the case with NP-hard problems. Each heuristic is developed to solve particular problem, and it is not applicable to other problems or some class of problems. On the other hand, metaheuristic is a method that can solve different classes of problems with small adaptations. Some of the best-known metaheuristics are Tabu search [4], Lagrangian relaxation [3], Variable neighborhood search [7] and a many metaheuristics inspired by nature. Particle Swarm Optimization (PSO) is metaheuristic inspired by nature introduced by Kennedy and Eberhart in [9].

MATHEMATICAL FORMULATION OF THE MCLP

The next model presents a mathematical formulation of the MCLP-a:

- P - the number of facilities,
- t_{ij} - the distance between locations *i* and *j*,
- R - covering radius,
- $Y_i \in \{0,1\}$ - denotes if some facility is located in location i,
- $X_i \in \{0,1\}\}$ - denotes if a location i is covered by some facility, that is $t_{ij} \leq \mathrm{R}$, for some facility j.

With given notation, the MCLP can be formulated as follows:
maximize the function

$$f = \sum_i X_i \tag{1}$$

with condition

$$\sum_i Y_i = P \tag{2}$$

METAHEURISTIC PSO

Particle Swarm Optimization (PSO) is population-based technique suitable for several optimization problems, introduced by Kennedy and Eberhart in [9], [11]. It is nature-inspired algorithm, inspired by the social behavior of individuals (particles) inside swarms in nature (e.g. flocks of birds or fishes). In each iteration, position of solution is updated using global best and own best position. Original PSO is applicable for optimization problems with continuous variables, but there are many adaptations of PSO method for application to discrete problems, known as Discrete Particle Swarm Optimization (DPSO) problem, proposed by Kennedy and Eberhart 1997 in [10]. In MCLP is finite number of locations for facilities, so for solving MCLP it is necessary to use the DPSO methos.

This method contains a swarm S with n particles $S = 1, 2, ..., n$ in d-dimenzional binary space. Each particle has own position $x_i = x_{i1}, x_{i2}, ..., x_{id}$ and velocity $v_i = v_{i1}, v_{i2}, ..., v_{id}$. The vector of position x_i represents a vector in the solution space and the velocity vector v_i determines the position of particle in the next iteration. With the given notation, position of particle in k iteration is calculated by formula $x_i^k = x_i^{k-1} + v_i^k$.

Each particle updates its velocity with its best position so far (b_i) and the best (g_i) with next equation, given in [11]:

$$v_i^k = c_1 \xi v_i^{k-1} + c_2 \xi (b_i - x_i^{k-1}) + c_3 \xi (g_i - x_i^{k-1}) \tag{3}$$

The parameters c_i represents the degrees of confidence of particle i in the different positions that influence its dynamics. The term ξ is a random number with uniform distribution $[0, 1]$ that independently generated in each iteration

Velocity v_{ij} determines the probability that the j-th binary variable of i-th particle, x_{ij} assumes a value of 0 or 1 at the next iteration. For assigning a new position value to a particle i, each position variable x_{ij} is randomly set with probability of selecting a value of 1 given by sigmoid function:

$$\frac{1}{1 + e^{-v_{ij}}} \tag{4}$$

Velocity term is limited with $|v_{ij}|<6$ [9]. This condition prevents the probability of the particle element assuming either a value 0 or 1 from being too high.

In the MCLP, position of each particle $x_i = x_{i1}, x_{i2}, ..., x_{id}$ has following properties:

- $x_{ij} \in \{0,1\}$
- $x_{ij} = 1$ if and only if facility placed on j-th location in of i-th particle.

In MCLP, number of facilities S is constant in each particle (condition 2), it is necessary to provide the sub-method for solving this requirement. Precisely, it is necessary to make corrections of solution in each iteration, so that each particle contains the same number P of locations for facilities.

COMPUTATIONAL TEST

Computational tests were based on method, instances and results given in [2]. Instances are available on http://www.lac.inpe.br/ lorena/instancias.html and they represent a real data collected at the central area of the Sao Jose dos Campos city (Brazil) using the Geographical Information System (GIS) ArcView. The problem was to find locations for antennas for Internet service with coverage radius of 800m

Three instances with 324, 402 and 500 locations were used in tests and the goal was to find locations for 5 antennas (in the first instance), 6 antennas (in the second) and 8 antennas (in the last one) with coverage of 100%. Each instance was attempted to be solved with swarms of 5, 10 and 20 particles in maximum 1000 iterations. The algorithm described is coded in C#.NET and the tests made on a computer with Intel i7-2670QM processor 2,2GHz and 8GB of RAM.

Like in [2], the results of best covers over 20 runs are presented in the Table 1. Number n represents number of location, P number of facilities (antennas), R radius of coverage, SS swarm size, and last three columns show percentage of covered locations, number of completed iterations and time to achieve a solution.

Table 1. Computational tests

N	*P*	*R*	*SS*	*cover* %	*Iter*	*time(ms)*
324	5	800	5	100	23	882
324	5	800	10	100	28	2124
324	5	800	20	100	13	1965
402	6	800	5	99,75	1000	15756
402	6	800	10	100	858	98982
402	6	800	20	100	136	31821
500	8	800	5	99,8	1000	7963
500	8	800	10	100	405	80934
500	8	800	20	100	639	238607

CONCLUSIONS AND FUTURE WORK

This paper shows a modification of PSO method for solving MCLP and computational tests confirm the effectiveness of this approach. A little swarms in small number of iterations solve the problem very fast. At the other hand, in the case of more complex problems, larger swarms in combinations with large number of iterations could solve it.

Future research in this field will go in two directions. The first direction will be to improve this approach by increasing a speed of convergence to solution with modifications a sub-method for corrections of particles on each iteration. The second part of research will investigate the role of fuzzy sets in improving maximal covering location problem [12].

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ABOUT THE AUTHORS

Darko Drakulić, MSc, Faculty of Philosophy, University of East Sarajevo, Bosnia and Herzegovina, E-mail: ddrakulic@ffuis.edu.ba

Miroslav Marić, PhD, Faculty of Mathematics, University of Belgrade, Serbia, E-mail: mmaric@matf.bg.ac.rs

Aleksandar Takači, PhD, Faculty of Technology, University of Novi Sad, Serbia, E-mail: stakaci@tf.uns.ac.rs

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