Integral model of vertical non-isothermal flat jets

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Abstract: *Over the outbreak of fire is occurs vertically flow of high temperature products primarily gaseous. Depending of the power of the fire this particles are "exported" to a considerable height and if there is a presence of wind they can reach the farm buildings, power lines and etc. In the current work is make an integrated model of such flow taking into account and lift force.*

Key words: *integral method, vertical non isothermal flow*

MATHEMATICAL MODEL OF THE FLOW

When there is a difference between the temperature of jet and environment $(T_{cm} \neq T_{ок})$, respectively $(\rho_{cm} \neq \rho_{ок})$ lift force (Archimedes force) is arise. For elementary section with thickness dx and surface f lift force is determine by the dependence:

$$Ar = -\left[\int_0^f (\rho_{cm} - \rho_{ок}) g df\right] dz \quad (1)$$

When $\rho < \rho_{ок}$ then the lift force acts upward in positive direction of the axis z, and that resultant jets is with positive lift force which leads to increase quality of movement.

Equation for Archimedes force and equation of the quality of movement of upward stream are assimilate and it is obtain:

$$-\left[\int_0^f (\rho - \rho_{ок}) g df\right] dx = d\int_0^f \rho u^2 df \quad (2)$$

It is divided equation (2) on the density of the environment $(\rho_{ок})$ and the gravity acceleration (g), then we have:

$$1 - \int_0^1 \frac{\rho}{\rho_{ок}} \frac{df}{f} = \frac{1}{gf} \frac{d}{dx}\left[u_m^2 f \int_0^1 \frac{\rho}{\rho_{ок}} \left(\frac{u}{u_m}\right)^2 \frac{df}{f}\right] \quad (3)$$

It is assumed that the pressure in the stream is not changed $(p = const.)$, then using of the Klaypeyron equation is obtained:

$$\frac{\rho}{\rho_{ок}} = \frac{T_{ок}}{T} = \frac{T_{ок}}{T_{ок} + \Delta T} = \frac{1}{1 + \frac{\Delta T}{\Delta T_m} \frac{\Delta T_m}{\Delta T_1} \frac{\Delta T_1}{T_{ок}}} \quad (4)$$

SYMBOL AND ACCEPTIONS

During solving the problem are import the following acceptions:

$$\theta_1 = \frac{T_1}{T_{ок}} \quad (5)$$

$$a = \frac{\Delta T_m}{\Delta T_1} \frac{\Delta T_1}{T_{ок}} = \frac{\Delta T_m}{\Delta T_1} (\theta - 1) \quad (6$$

where $T_{ок}$ и T_1 are the temperatures of the environment and jet in his initial cross section. Parameter a is constant for the given cross section of the jet.

The value of the integrals are receive for flat and axis symmetrical jets are according [1],[2]:

$$\frac{\Delta T}{\Delta T_m} = 1 - \left(\frac{y}{b}\right)^{3/2}$$

$$\frac{u}{u_m} = \left[1 - \left(\frac{y}{b}\right)^{3/2}\right]^2 \qquad (7)$$

Because it is considering infinity flat jet for its face are obtain:

$$f = 1.b = b \qquad (8$$

where b is width of the jet.

For axis symmetrical jets the face is given by the equation:

$$f = \pi r^2 \qquad (9)$$

After so make assumptions for (3) is receiving the equation for the quantity of motion for a vertical non-isothermal jet:

$$1 - A_0 = \frac{1}{gf}\frac{d}{dx}\left(u_m^2 f A_2\right) \qquad (10)$$

In particular solution of the problem are put values which will be integrate for flat and axis symmetric jet according Table 1 and also for f according equations (8) and (9).

Table 1

	Basic case	Flat jet	Axis symmetrical jets
A_0	$\int_0^1 \frac{1}{f\left(1 + a\frac{\Delta T}{\Delta T_m}\right)} df$	$\frac{1}{1+0,77a}$	$\frac{1}{1+0,307a}$
A_2	$\int_0^1 \frac{\left(\frac{u}{u_m}\right)^2}{\left(1 + a\frac{\Delta T}{\Delta T_m}\right) f} df$	$\frac{0,316}{1+0,865a}$	$\frac{1}{1+0,77a}$

Unknown values, which are receive are necessary for the description of stream are: maximum velocity, temperature difference and the width of the jet. This requires three integral conditions for their determination. The possible options are:

- The equation for the quality of movement (pulse); equation for the total energy and heat content
- The equation for the quantity of movement to maintain the heat content and the semi-empirical dependences for the width of the jet

The first option requires a comparatively difficult decision with a compression of the program for the numerical solution of a system of three ordinary differential equations.

In the second case, the problem is alleviate significantly by reducing the number of differential equations, and for the width of the flow boundary layer are using the expression [1]:

$$\frac{db}{dx} = 0,22\frac{\rho_m + \rho_{ок}}{2\rho_{ок}} \qquad (11)$$

According to (6) for a follows:

$$\frac{db}{dx} = 0,22\left(1 + 0,5a\right) \qquad (12)$$

The equation for the conservation of the heat content in the jet using initial value for ΔQ_0 is determine by the expression:

$$\Delta Q_0 = \int_0^f c_p \rho g u \Delta T df = \Delta Q = const. \tag{13}$$

Equation (3) is divide on $c_p \rho_{ок} g = const.$, and infront of integral is carry out ΔT_m and u_m:

$$\frac{\Delta Q_0}{\rho_p \rho_{ok}} = f u_m \Delta T_m \int_0^1 \frac{\rho}{\rho_{ok}} \frac{u}{u_m} \frac{\Delta T}{\Delta T_m} \frac{df}{f} \tag{14}$$

In the right side of equation14 integrals are process in the form:

$$B_2 = \int_0^1 \frac{\frac{u}{u_m} \frac{\Delta T}{\Delta T_m}}{1 + a \frac{\Delta T}{\Delta T_m}} \frac{df}{f} \tag{15}$$

With accepted above dependencies for dimensionless cross section of distribution for velocity $\frac{u}{u_m}$ and the ratio of the temperature difference $\frac{\Delta T}{\Delta T_m}$ according to (7), (8) and (9) is received values shown in Table 2

Table 2

	Flat jet	Axis symmetric jet
B_2	$\frac{0,368}{1+0,837a}$	$\frac{0,18}{1+0,732a}$

Equation (14) take the form:

$$f u_m a = \frac{\Delta Q_0}{c_p \rho_{ok} g T_{ok} B_2} \tag{16}$$

respectively

$$u_m = \frac{\Delta Q_0}{c_p \rho_{ok} g T_{ok} B_2 a f} \tag{17}$$

From (17) is evident that the attenuation of the maximum velocity of the upward non isothermal flow is proportional to the power of its source. This "source" with power ΔQ_0 can be heated surface, outbreak of fire or etc. It is inversely proportional to the density ρ_{ok} and the ambient temperature T_{ok}. As these two factors are related $\rho_{ok} = \frac{p}{RT_{ok}}$ and had be sought for appropriate balance between them. In this case parameter a is decisive which contains the ratio of the temperature in the jet and in environment. If a is reduce that will lead to a slower decay of u_m.

FLAT VERTICAL NON-ISOTHERMAL JETS

This decision is base on received above equations for conservation of the quantity of movement of the heat content (10) and (16). They are process using the accepted dependencies for the section area $(f = 1.b)$, and after the appropriate processing is obtain:

$$\frac{0,485abg}{1+0,47a} = \frac{d}{dx}\left(u_m^2 b \frac{1}{1+0,865a}\right) \tag{18}$$

From the equation to conversation of the heat content decide regarding on parameter a follows:

$$a = \frac{\Delta Q_0 \left(1 + 0,837a\right)}{0,368 c_p \rho_{ok} g T_{ok} b u_m} \tag{19}$$

In equation 18 a is replace:

$$\frac{0,2abg}{1+0,47a} = \frac{\Delta Q_0}{\left(c_p \rho_{ok} g T_{ok}\right)^2} \frac{d}{dx}\left[\frac{\left(1+0,837a\right)^2}{ba^2\left(1+0,865a\right)}\right] \tag{20}$$

From equation 12 can be get the following connection:

$$dx = db\left[0,22\left(1+0,5a\right)\right]^{-1} \tag{21}$$

dx with are replace db according to (21) in equation 20:

$$\frac{0,915abg}{\left(1+0,47a\right)\left(1+0,5a\right)} = \frac{\Delta Q_0^2}{\left(c_p \rho_{ok} T_{ok}\right)^2} \frac{d}{db}\left[\frac{\left(1+0,837a\right)^2}{ba\left(1+0.865a\right)}\right] \tag{22}$$

as

$$\frac{\Delta Q_0^2}{\left(c_p \rho_{ok} T_{ok}\right)^2 g} = const. = L_1 \tag{23}$$

Equation 22 can be write as:

$$\frac{0,915ab}{\left(1+0,47a\right)\left(1+0,5a\right)} = L_1 \frac{d}{db}\left[\frac{\left(1+0,837a\right)^2}{ba^2\left(1+0,825a\right)}\right] \tag{24}$$

From equation 24 can be determine the expansion of the flat non-isothermal turbulent jet. Then after this equation is substitute into equation 19 allows the determination of the attenuation of the maximum temperature difference ΔT_m. With this value for $\theta_1 = \frac{T_m}{T_{ok}}$ is solve equation 18, thereby is obtaining the maximum temperature in dependence of the height x.

The receiving system of ordinary differential equations is necessary to be solve numerically with appropriate software.

GN Abramovich [1] was made the following approximate solution: Considering "weak" non-isothermal medium at which the relation $\theta_1 = \frac{T_m}{T_{ok}}$ leads to:

$$a = \left(\theta_1 - 1\right)\frac{\Delta T_m}{\Delta T} = const. \tag{25}$$

This means that he consider a isothermal flow. As a result of the decision of this simplified version is obtain:

$$u_m = \sqrt[3]{2q_n + \left(u_0^3 - 2q_n\right)\left(\frac{b_0}{b}\right)^{1,5}} \tag{26}$$

$$\Delta T_m = \frac{0,3q_n}{gb\left[2q_n + \left(u_0^3 - 2q_n\right)\left(\frac{b_0}{b}\right)^{1.5}\right]^{0.333}} \tag{27}$$

where $q_n = \frac{0,15Q_0}{c_p \rho_{ok} T_{ok}}, m^3/s^3$; u_0, b_0 - velocity and width at given closed to the initial cross section.

Parameter a can be define with expression:

$$a = \frac{\Delta T_m}{T_{ok}}$$

If the equation above is use it can be determinate b as follows:

$$\frac{db}{da} = 0,22\left(1+0,5a\right) \quad (28)$$

The decision has very approximate character because it does not account that the flow is non-isothermal assuming $T_0 \approx T_{ок}$. This requires a different and relatively more accurate approach to solve the problem

CONCLUSION

Integrated solution which is receive for a flat vertical non-isothermal jet allows a relatively fast and accurate analysis of the flow using variation of large number of initial parameters. As results are obtain integral parameters u_m, ΔT_m and b, which describe the distribution of the jet.

LITERATURE

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Paper is reviewed.