Vibration isolation experimental setup. Part II: Theoretical investigation

Svetlin Stoyanov

Vibration isolation experimental setup – theoretical investigation: *The paper presents a parameter identification of a mechanical model, which describes an experimental setup for vibration isolation studding. The logarithmic decay and the damping coefficient are obtained from the free response in the time domain. A stiffness coefficient is obtained from frequency domain of the free response. A theoretical simulation is realized and the parameter's values are verified in the frequency domain.*

Key words: *vibrations, vibration isolation, parameters identification, signal processing, experimental setup.*

1. Introduction

An experimental setup for studying of mechanical vibrations is presented in [6]. With the help of this setup, the resonance phenomenon can be demonstrated. Also, the amplification of the amplitude of vibrations as well as the vibration isolation can be observed. To understand the relationships between these experimentally observed processes and the theory of mechanical vibrations, it is necessary to do a theoretical investigation. This means to derive and solve the differential equation of the vibrations. It will give a solution in the time domain. After that, as a general rule, it is necessary a Discrete-time Fourier Transform (DFT) to be done for transferring the time-diagram to spectrogram of the vibrations [4, 5]. In this work, the vibrations are harmonic and it is no need to take DFT. It is enough just to find the maximum value of a steady state time signal and it is the amplitude of the vibrations for given frequency.

The aim of this work is to identify the parameters of the mechanical model of the experimental setup, which is presented in [6]. Also, this work considers a theoretical simulation of this experimental setup. Another one aim of this paper is to compare and correlate the models used in the classical mechanics and in the control theory. Also, this paper is concerned about comparing and correlating the notations of the model parameters. In the Russian-language literature, it is appropriate to notate the stiffness coefficient with c and the coefficient of viscous friction with k. Contrariwise, in English literature, it is appropriate to notate the stiffness coefficient with k and the coefficient of viscous friction with c. In [6] is used the English notations. In this work is used the Russian notations.

2. Mechanical model

As it is justified in [6], the mechanical model for investigation of the vibrations of the experimental setup is one DOF with kinematic excitation. This model is shown on Fig. 1. In this work, the excitation is harmonic, so $x_0 = A_0 sin(pt)$.

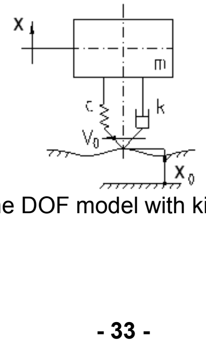

Fig. 1 One DOF model with kinematic excitation

3. Mathematical models

The differential equation of the investigated vibrations is

$$m\frac{d^2}{dt^2}x(t) + c\frac{d}{dt}x(t) + kx(t) = A_0\, c\, sin(pt) + A_0\, k\, cos(pt). \quad (1)$$

3.1. Mathematic model, commonly used in the classical mechanics

In the classical mechanics, Eq. 1 is usually written as [2, 7]

$$\frac{d^2}{dt^2}x(t) + 2n\frac{d}{dt}x(t) + \omega^2 x(t) = F_0 sin(pt - \varphi_0), \quad (2)$$

where

$$\omega = \sqrt{\frac{c}{m}} \quad (3)$$

is the natural frequency of the system and

$$n = \frac{k}{2m}\,. \quad (4)$$

is the damping coefficient.

Equation (2) has a harmonic steady state solution:

$$x = Asin(pt - \varphi). \quad (5)$$

The parameters of Eq. 5 are obtained through substituting of this equation and its derivatives in the differential equation. Then, the amplitude is

$$A = A_0\frac{1}{\sqrt{\left(1-\frac{p^2}{\omega^2}\right)^2 + 4\frac{n^2p^2}{\omega^4}}}. \quad (6)$$

3.2. Transfer function

In the control theory, Eq. 1 is usually written as [1,3]

$$T^2\frac{d^2}{dt^2}x(t) + 2\xi T\frac{d}{dt}x(t) + x(t) = F_0 sin(pt - \varphi_0), \quad (7)$$

where

$$T = \frac{1}{\omega} \quad (8)$$

is the time constant of the system and

$$\xi = \frac{1}{2}\frac{k}{\sqrt{mc}} \quad (9)$$

is the damping ratio.

The transfer function of mechanical system investigated is derived from Eq. (7) with the help of the Laplace transform and has the following forms [1, 3]:

$$T(f) = \frac{\omega^2}{T^2 f^2 + 2\zeta T f + 1}. \quad (10)$$

or equivalently

$$T(f) = \frac{A_0\omega^2}{f^2 + 2\zeta\omega f + \omega^2}. \quad (11)$$

4. Model parameter identification

The parameters of this model are: the mass m, the stiffness coefficient c, and the viscous friction coefficient k. The mass value is measured, and in this work it is $m = 0.42\ kg$,

4.1. Estimating the damping from the time domain of the free vibrations

The time-diagram of the free vibrations of the system investigated is shown on Fig. 2. The blue colored line represents the experimentally obtained data. The logarithmic decay is:

$$\delta = \frac{1}{N} ln\left(\frac{a_i}{a_{i+N}}\right) \quad (12).$$

For number of periods $N = 2$ and for the corresponding acceleration amplitudes $a_1 = 19.8\, m/s\text{^}2$ and $a_3 = 8\, m/s\text{^}2$, it is obtained $\delta = 0.453$.

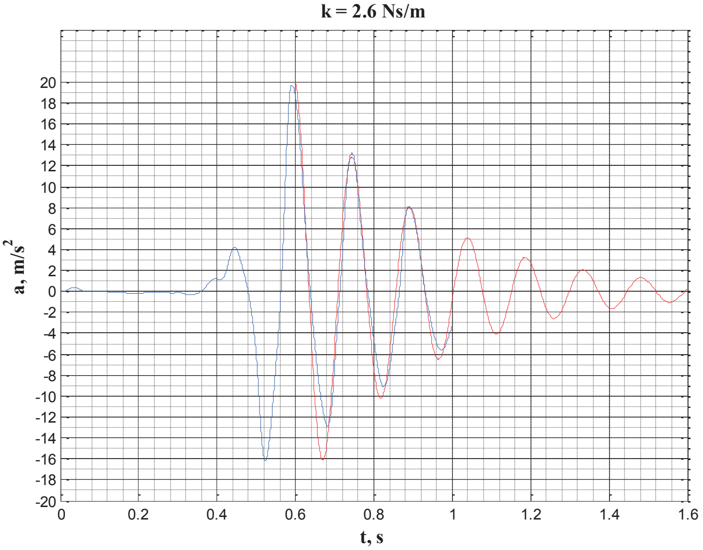

Fig. 2 Free response of the investigated system

The equation for the damping coefficient is:

$$n = \delta \frac{p}{2\pi}. \quad (13)$$

According to experimental investigation in [6], the resonant frequency is $f = 6.8\, Hz$. Therefore, the resonant circular frequency, it $p = 13.6\,\pi\, rad$. Then, according to Eq. 13, the damping coefficient is $n = 3.08\, rad/s$. The last step is to calculate the coefficient of the viscous friction from Eq. 4:

$$k = 2\, m\, n. \quad (14)$$

From this equation, it is obtained $k = \ 2.6\, Ns/m$.

On Fig. 2, the red colored line represents a theoretically obtained free response. It is gathered from numerical solving of Eq. 1 for $A_0 = 0$ and non-zero initial displacement. This theoretical solution is for k = 2.6 Ns/m, and one can observe that the free response calculated with this value fits to the tree response, which is obtained experimentally.

4.2. Estimating the value of the stiffness coefficient

From the roots of the characteristic equation of the Eq. 2, it is follows that the frequency of the free response is

$$\upsilon = \sqrt{n^2 - \omega^2}. \quad (15).$$

The resonant circular frequency p is the same as the frequency of the free response υ. Also, the free response frequency υ, can be obtained from the spectrogram of the free response, which time-diagram is already shown on Fig. 2. After discrete-time Fourier transform, the spectrogram is obtained and presented on Fig. 3. One can observe that the value from the spectrogram $f = 6.84\,Hz$ is close to the resonant frequency $f_r = 6.84\,Hz$, wcis id determined in [6].

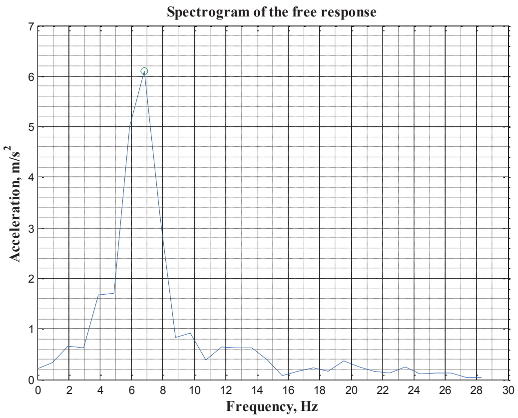

Fig. 3 Spectrogram of the experimentally obtained system free response

One can obtain the natural frequency from Eq. 15 as follows:

$$\omega = \sqrt{n^2 - p^2}. \qquad (16)$$

According to Eq. 16 and Eq. 3, the stiffness coefficient of the spring is $c = 763\ N/m$.

4.3. Simulating and verifying in the frequency domain of the forced vibrations

The experimentally obtained graph of amplitude versus frequency [6] is shown with green line on Fig. 4.

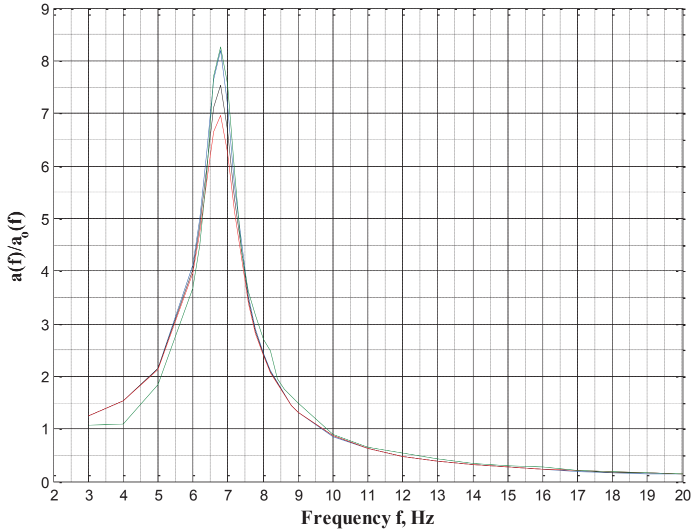

Fig. 4 Amplitude versus frequency graphs

In this work, a numerical solution of Eq. 1 is obtained through function ode45 of the software system Matlab. This function implements the Runge-Kutta method [8]. The maximal value of steady state acceleration is found and it is the amplitude for the corresponding frequency. In this way, the theoretical graph of amplitude versus frequency is obtained and plotted also on Fig 4. The red line corresponds to k = 2.6 Ns/m. A difference in the magnitudes at the resonance can be observed. Therefore, the theoretical graph is calculated and plotted also for k = 2.4 Ns/m (black line) and k = 2.2 Ns/m (blue line). Among these three values, the best fit is for k = 2.2 Ns/m.

One can observe that the resonance point of the graph of amplitude versus frequency is significantly more sensitive to damping than the free vibrations decay. It is clearly seen by comparing Fig. 2 and Fig. 5. On these figures, it is to be observed that the free vibrations decay is slightly influenced when the viscous friction coefficient k changes from 2.2 Ns/m to 2.6 Ns/m.

On Fig. 4, one can observe that the magnitudes, which corresponds to the low frequencies of 3Hz and 5 Hz has invalid. This is due to insufficient accuracy of the accelerometers used.

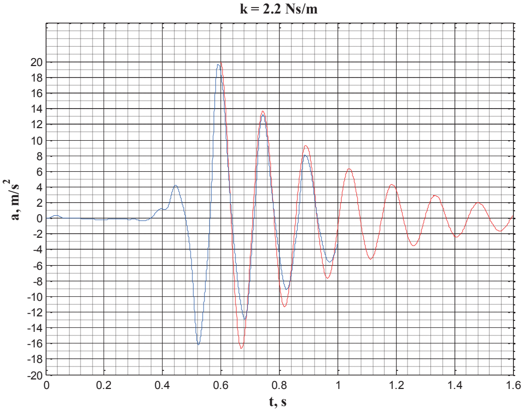

Fig. 5 Free response comparison

ЛИТЕРАТУРА

[1] Charles M. Close, Dean K. Frederick, Jonathan C. Newell. Modeling and analysis of dynamic systems. Jonh Wilew & Sons, Inc., 2002.

[2] Dresig, H., F. Holzweibig. Dynamics of Machinery. Theory with applications. Springer, 2010.

[3] Lehov, G. Control theory. University of Ruse, ISBN 978-954-712570-4, 2012. (In Bulgarian.)

[4] Stoyanov, Sv. Scientific Software Tools for Machinery Vibration Diagnostic. Proceedings of International Conference on Computer Systems and Technologies - CompSysTech'13, Ruse, 2013.

[5] Stoyanov, Sv. Time and frequency domain properties of free vibrations, caused by a backlash. Mechanics of machinery, National committee of theoretical and applied mechanics, Technical University of Varna, 2013, ISSN 0861-9727 (In Bulgarian)

[6] Stoyanov, Sv. Vibration isolation experimental setup – part I: Experimental setup description Proceedings of University of Ruse, ISSN 1311-3321, Mechanics, Mechanical and Manufacturing Engineering, Ruse, 2014 (current issue).

[7] Thomson, W. Theory of vibration with applications. Chapman & Hall, 1993.

[8] Veleva, E., S. Karakoleva. Numerical methods and statistic – theory and practice with Matlab. ISBN 954-712-245-2, University of Ruse, Ruse, 2011За контакти: PhD Eng. Svetlin Petrov Stoyanov, sstoyanov@uni-ruse.bg, Department of Technical Mechanics, University of Ruse, BG-7017 .

Докладът е рецензиран.