Mathematical Problem Posing on the Basis of the GeoGebra Multi-platform

Magdalena Petkova, Emiliya Velikova

Abstract: Mathematical Problem Posing on the Basis of the GeoGebra Multi-platform. *The main goal of this article is to present some opportunities for professional competency development and stimulate the creativity of students in Mathematics and Informatics by mathematical problem posing and problem solving and constructing them into a dynamic environment. The possibilities of the online multi-platform GeoGebra as a basis for the process of professional competence development and stimulate the creativity of future teachers of mathematics, have been analysed.*

Keywords: *mathematical problem posing, GeoGebra multi-platform, professional competency development, creativity*

INTRODUCTION

Targeted use of technology in the educational process determines discussions:

1. *on the requirements for new strategies and techniques to ensure mastering of knowledge and skills in learners to adapt to new technological environments;*
2. *on public expectations for schools and higher education* for the preparation of "... flexible and adaptable young people; ambitious and disruptive personalities; people with social communication skills, presentation skills, critical thinking, teamwork, ability to learn quickly; individuals with skills to work with information - searching, collecting, processing and presentation" [2, p. 96];
3. *on the need for professional competence development of teaching staff,* such as continuous update of existing knowledge, new teaching methods' learning, training to use information and communication technologies [1], etc.;
4. *on problems in the professional competence development of students - future teachers of mathematics,* such as the learning process, the polar attitude of students towards innovations during the learning process, practical training of students, the state of private didactics, differences in the understanding of the goals and values of education [4, p. 38].

These problems do not support the formation of qualified teaching staff who could skilfully solve the current existing problems of education. The main purpose of this article is an analysis of:

1. the theoretical research on the possibilities of professional competence development and to stimulation of the creativity of students - future teachers of mathematics by mathematical problem posing and problem solving, and constructing those into a dynamic environment;
2. the possibilities of the GeoGebra online multi-platform as a basis for the process of professional competence development and creativity of students in *Mathematics and Informatics*.

PROFESSIONAL COMPETENCE AND CREATIVITY DEVELOPMENT BY MATHEMATICAL PROBLEM POSING AND PROBLEM SOLVING

What are the opportunities for professional competence development and stimulation of the creativity of students - future teachers of mathematics by mathematical problem posing and problem solving?

Problem posing commonly defined as:

1. engaging a person in a specific task, the goal of which is to generate a new problem with a specific system of conditions [26, p. 19];
2. generating new problems and reformulating of existing ones [8], [29, p. 3];

3. a process by which, based on mathematical experience, students construct personal interpretations of concrete situations and from these situations formulate meaningful mathematical problems [31, p. 518], [34, p. 9];
4. a special case of problem solving [19, p. 151], etc.

The most common strategies, used for mathematical problem posing, are:

1. *Changing the constraints* [28]*;*
2. *Generalization* [3, p. 9], [10]*;*
3. *Posing additional/ancillary problems* [30]*;*
4. *Analogy; Association* [3, p. 16], [17]*;*
5. *Translation* [21, p. 517];
6. *Substitutions* [3, p. 13]*,* etc.

Many researchers consider problem posing as an integral part of the process of problem solving [6, p. 3]. It is experimentally confirmed that the problem solving and the ability to pose problems correlate and thus the problem posing positively affects the problems' solving problems [18, p. 162].

The qualities of problem-solving activities that occur under effective implementation are *skill and ability to solve problems in different specific ratios* [4, p. 9].

General criteria for evaluation of the process of creating the tasks offered by E. Silver and J. Cai: *quantity, originality and complexity* [27]*.*

I. Kontorovich, B. Koichu, R. Leikin and A. Berman use the other three criteria for evaluating the process of problem posing when working in small groups, *fluency, flexibility and originality* [20, p. 121]. With the criteria *consideration of aptness,* group's work is assessed as a team one or not. The group of researchers reached the conclusion that "Considerations for aptness take into account the process of creating tasks, but not only its final results." [20, p. 124].

The problem posing in small groups can be seen as a *creative activity* the assessment of which is based on quantitative assessment of:

1. *the fluidity of the thinking* – number of created mathematical problems that meet the requirements of the posed problem;
2. *flexibility of the thinking* - number of different types of mathematical problems and number of implemented strategies for problem posing;
3. *originality of the thinking* - number of created mathematical problems posed by very few or no other people [20, p. 124].

X. Yuan and B. Sriraman integrate the three criteria *fluency, flexibility and originality* of the thinking in creative thinking tests [34].

G. Polya evaluates the complex processes of thinking to solve mathematical problems with the following criteria: *understanding the condition of the problem, planning a strategy for solving, finding solutions and looking back* [25, pp. xvi–xvii].

Therefore, skills for mathematical problem posing and problem solving can be regarded as a component of creative development. K. Kojima, K. Miwa and T. Matsui underline the connectivity between the problem posing and the creativity that describes the problem posing has an aspect of creativity because it requires productive thinking [18, p. 164].

P. Petrov determines the solving of problems that require an original way of solving as feasible in situations, which are appropriately organised for creative solutions. These situations should [4, p. 37]:

1. allow multivariate search for a solution;
2. give a research nature to the activity;
3. combine common sense solution and compilation of mathematical problems, along with a search for an equilibrium between the processes of forecasting and controlling.

The organisation of the mental activity of the learner to discover one's unconventional views, original solutions to problems and tasks depend on [4, p. 38]:

1. the role of the trainer who has the ability to solve problems;
2. the creation of an environment for the implementation of an imitative action by practice (exercises);
3. the reflexive attitude of the learner towards the activities of searching for solutions and developing of skills as an introspection of their experience;
4. the establishment of an effective organisation of the mental activity of students in problem solving;
5. the values of the trainee's in the selection of tasks - priority and ease of orientation into the problems.

A scientific literature research confirms that the skills of posing and solving mathematical problems are among the main elements of the professional competences of future mathematics teachers.

M. Ticha and A. Hospesova describe the problem posing as one of the attributes of the subject didactic competence of the teacher [33, p. 133].

R. Vasileva-Ivanova analyses that the competence to solve problems, which is from the group of mathematical competences is a component of the professional competence of future mathematics teachers [1, p. 182].

P. Petrov noted that the development and application of a typical system of concepts and theoretical models that develops the main components and overall structure of the ability to solve problems, provides an opportunity to build variants of effective strategies for training of teachers, training of students and pupils in the context of portable competence [4, p. 27].

Therefore, the development of skills for posing and solving mathematical problems is the basis for the development of professional competence of future pedagogical staff.

POSSIBILITIES OF GEOGEBRA ONLINE MULTI-PLATFORM

The GeoGebra online multi-platform (version: 10 October 2015) is part of a long-standing research of the *International Institute GeoGebra* [11]. The GeoGebra project starts with the creation of a computer program, which the main function is to connect the principles of mathematical education with the principles of use of computer technologies in the educational process. Over the years, the project develops and expands to comply with to the modern educational needs. Today, the multi-platform covers five lines: *GeoGebra software, GeoGebraForum, GeoGebraTube, GeoGebraBook and GeoGebraGroup*. The access to it is distributed according to the needs of users who may or may not have created a profile.

The components of the GeoGebra multi-platform are:

1. *GeoGebra, a dynamic mathematical software,* which has the following basic characteristics:
   - *two operating modes:* online (Web App, [12], [16]) and offline [12];
   - *compatibility with operating systems for computer configurations* - Chrome App, Windows, Mac OS X; Linux (Red Hat, openSUSE, Debian, Ubuntu); for tablets in Windows Store, Apple's App Store, Android's Google Play); for phones (under testing process);
   - *an easy to use interface, multilingual menu, commands* [9, p. 192];
   - *dual representation of objects:* every expression in the algebraic window corresponds with one in the geometry window and back [23, p. 146], etc.
2. *GeoGebraForum* [13] - exchange ideas, share problems, etc.
3. *GeoGebraTube* [15] – a basis for creating, searching, publishing and sharing of online material.
4. *GeoGebraBook* –
   - allows the integration of *text, video, applets, images, web addresses, grouping materials* (GeoGebraWorkSheets);

- ensures conditions for *asking questions and creating exercises*, active only in a test version.

5. *GeoGebraGroup* [14] - a new approach to the organisation of online educational resources, which is in a process of improvement.

Some of the main advantages of the GeoGebra multi-platform in the context of the teaching of mathematics are that it offers opportunities for:

1. *trace ability of mathematical connections, relationships, patterns* between the sites participating in GeoGebra applets;
2. *simulations to analyse possible solutions* [23, p. 146];
3. *researching* by combining facts experimenting with certain combinations, analysing the results and raising hypotheses, creating models, confirmation or rejection of the hypothesis [23, p. 146];
4. *expanding and enriching learning environment* through the use of interactive methods and strategies of teaching and learning [22, p. 136];
5. *improving the existing static forms* of training materials *and implementing dynamic (multimedia) developments,* integrating image, sound, animation and text [22, p. 136];
6. *cooperative learning* [9, p. 192];
7. *mathematical problem posing and problem solving* [24], *logical proofs and geometric structures;*
8. *individual work;*
9. *mathematical research;*
10. *experimentation;*
11. *acquiring new knowledge and skills to work with the platform;*
12. *feedback;*
13. *summarising the problems and relationships* [7, p. 127];
14. *creating a learning environment;* etc.

The increasing use of Dynamic Geometry Software, as GeoGebra, provides conditions for the creation of new problems and their use in different contexts [7, p. 125].

Based on the concept of E. Stoyanova [32] for problem posing in conditions of *free, semi-structured and structured* learning situations, methods of teaching mathematics by M. Petkova and E. Velikova, have been developed[24].

Some of the suggested training methods based on the multi-platform are:

1. solving mathematical problems with GeoGebra tools;
2. solving problems by constructing or substantiating of claims in GeoGebra environment;
3. posing new problems by the possibilities of GeoGebra;
4. creating exciting problems from the perspective of the learner and in accordance with the studied subjects;
5. developing the skills of posing and solving problems in the GeoGebra environment.

It can be concluded that the use of the possibilities of the GeoGebra multi-platform for mathematical problem posing and problem solving, stimulates the creativity of future teachers of mathematics and supports the development of their professional competency.

MATHEMATICAL PROBLEM POSING AND PROBLEM SOLVING ON THE BASIS OF THE GEOGEBRA MULTI-PLATFORM

At the *GeoGebra Global Gathering*, 2015, we presented a methodology for developing mathematical problem-posing skills, problem-solving skills, and dynamic GeoGebra construction skills, using the *dynamic software GeoGebra* and *GeoGebraBook* (Figure 1, Figure 2) [24].

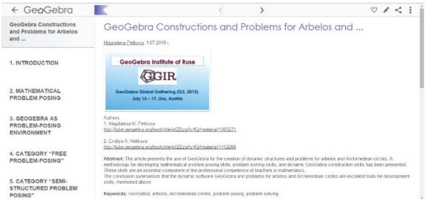

Figure 1. First GeoGebraBook (in English)
http://www.geogebra.org/material/simple/id/1383213

Figure 2. First GeoGebraBook (in Bulgarian)
http://www.geogebra.org/book/title/id/Y09YJ0Zc

The established by GeoGebraGroup, *"Math PP GeoGebra"* contains a theoretical-methodological material (Figure 3), examples of solved problems, problems for developing mathematical problem-posing skills, problem-solving skills (Figure 4) and questions with an open response created on the topic Arbelos and Archimedean circles [24].

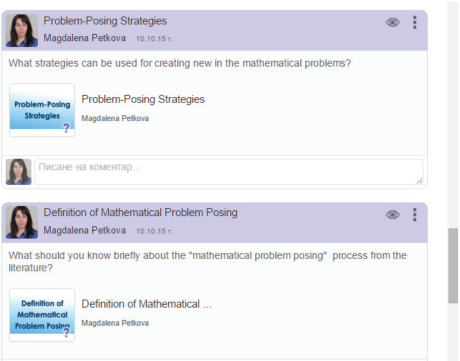

Figure 3. Theoretical-methodical materials published in GeoGebraGroup "*Math P-P GeoGebra*"

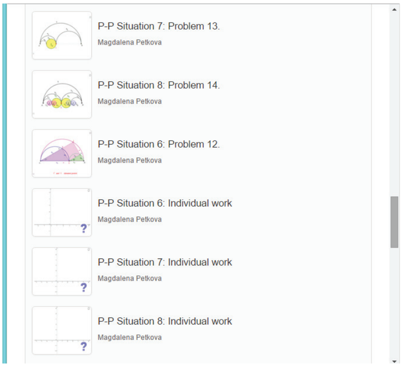

Figure 4. *A structured situations for creating mathematical problems* and problems for developing skills to create mathematical problem posing skills

The main elements that can contain different types of educational situations (*free, semi-structured*, *structured*) through which mathematical problem posing and problem solving skills can be developed (Figure 5, Figure 6), are:

1. a type of learning situation for creating mathematical problems;
2. condition and/or solution of a mathematical problem, part of a condition of the problem, and others;
3. GeoGebra dynamic structure for experimentation if needed;
4. opportunities to use fields for entering text/solution/response; a self-assessment button, when and if necessary; a button for saving and exiting;
5. GeoGebra fieldwork to illustrate the ideas of the learner;
6. questions with a selectable response.

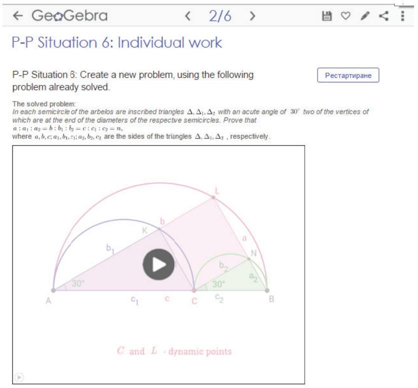

Figure 5. Elements of one educational situation

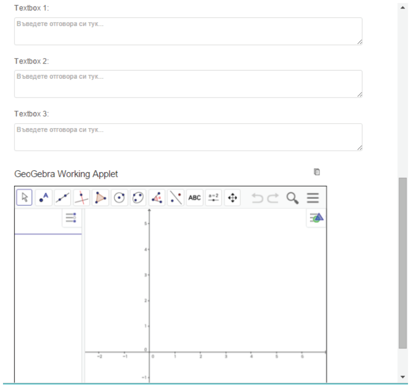

Figure 6. Elements of one educational situation

Here we present a methodology to use structured situations in order to form problem posing skills in future teachers of mathematics on the basis of the GeoGebra multi-platform.The strategies used in educational situations are:

1. "Analogy" - Problems 1 ÷ 4;
2. "Generalization" - Problem 5;
3. "Changing the constraints" - Problem 6. It shows the application of the "changing the constraints" strategy on Problem 1.

The presented problems and future developments will be included in the so-called section "School problems" (Enter ZPOPK on www.geogebra.org/groups to join. Use you private GeoGebra account).

P-P Situation "School problem": Create a new problem, using the following problem already solved.

"The School problem" [5, p. 78]: In a circle with a radius R three identical circles $k_1(A,r), k_2(B,r), k_3(C,r)$ are entered, each is tangent to the other two (Figure 7). Calculate the radius r.

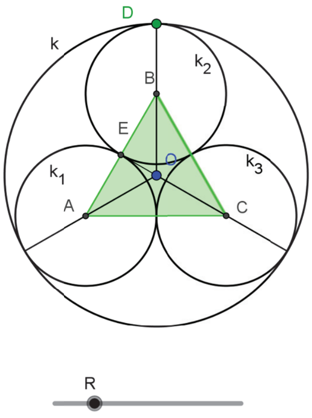

Figure 7

Solution: The points A, B, C are vertices of an equilateral triangle $\triangle ABC$.

$OD = R, AB = BC = AC = 2r.$

Let the point D be the tangent point between the greater circle k and one of the smallest circle, such as k_2.

$OE \perp AB, \sphericalangle EBO = 30°, BE = BD = r.$

$BO = DB + BO.$

In $\triangle OBE$: $EB = OB\cos(\sphericalangle EBO)$, then

$$OB = \frac{EB}{\cos(\sphericalangle EBO)} = \frac{r}{\cos 30°} = r : \frac{\sqrt{3}}{2} = \frac{2r}{\sqrt{3}},$$

from where it follows that

$$R = DB + BO = r + \frac{2r}{\sqrt{3}} = r\left(1 + \frac{2}{\sqrt{3}}\right).$$

The searched radius, expressed by the given radius R, is obtained by the formula:

$$r = \frac{R\sqrt{3}}{2 + \sqrt{3}}.$$

Problem 1 [5, p. 79]**:** In a circle with a radius R four identical circles $k_1(A,r), k_2(B,r), k_3(C,r), k_4(F,r)$ are entered, each is tangent to the other three (**Figure 8**). Calculate the radius r.

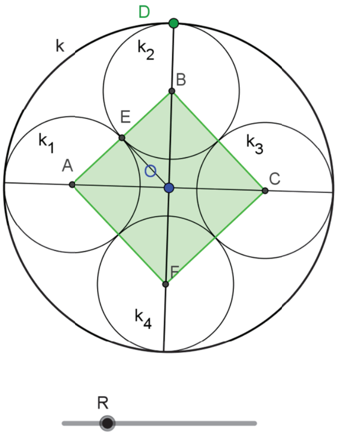

Figure 8

Solution: The points A,B,C,F are vertices of a square $ABCF$.

Let the point D be the tangent point between the greater circle k and one of the smallest circle, such as k_2.

$OD = R, BD = BE = r, OE \perp AB, AB = 2r.$

$OD = DB + BO,$ но $BO = r\sqrt{2}$ because the segment BO is half of the square's diagonal BC.

$$OD = DB + r\sqrt{2},$$

from where it follows that

$$R = r + r\sqrt{2} = r\left(1 + \sqrt{2}\right).$$

The searched radius, expressed by the given radius R, is obtained by the formula:

$$r = \frac{R(1-\sqrt{2})}{1+\sqrt{2}}.$$

After rationalizing, the denominator follows:

$$r = R\left(\sqrt{2} - 1\right).$$

Problem 2: In a circle with a radius R five identical circles $k_1(A,r), k_2(B,r), k_3(C,r), k_4(F,r), k_5(G,r)$ are entered, each is tangent to the other four (**Figure 9**). Calculate the radius r.

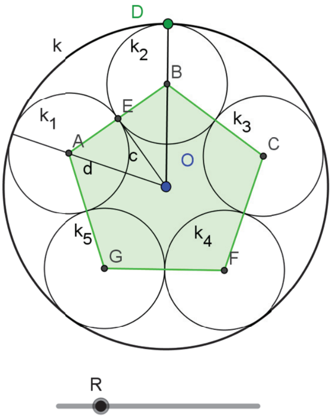

Figure 9

Solution: The points A,B,C,F,G are vertices of the regular pentagon $ABCFG$.

$OD = R, BD = EB = r, OE \perp AB, AB = 2r.$

$OD = DB + BO \Leftrightarrow R = r + BO.$

From the properties of the regular pentagon, that his side is equal to the product of half the radius of the circumscribed circle and $\sqrt{10 - 2\sqrt{5}}$ follows that:

$$AB = \frac{BO}{2}\sqrt{10 - 2\sqrt{5}}$$

$$2r = \frac{BO}{2}\sqrt{10 - 2\sqrt{5}}$$

$$BO = \frac{4r}{\sqrt{10 - 2\sqrt{5}}}$$

$$R = r + \frac{4r}{\sqrt{10 - 2\sqrt{5}}}$$

The searched radius, expressed by the given radius R, is obtained by the formula:

$$r = \frac{R\sqrt{10 - 2\sqrt{5}}}{4 + \sqrt{10 - 2\sqrt{5}}}.$$

Problem 3 [5, p. 79]**:** In a circle with a radius R six identical circles $k_1(A,r), k_2(B,r), k_3(C,r), k_4(F,r), k_5(G,r), k_6(H,r)$ are entered, each is tangent to the other five(**Figure 10**). Calculate the radius r.

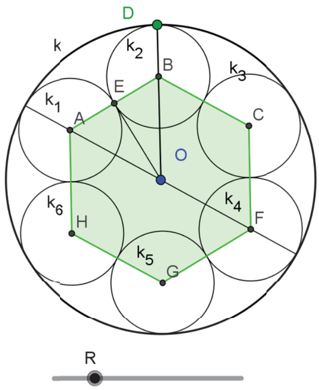

Figure 10

Solution: The points A, B, C, F, G, H are the vertices of the regular hexagon $ABCFGH$.

$OD = R, BD = EB = r, OE \perp AB$.

$AB = OB = 2r$ because the triangle $\triangle AOB$ is an equilateral triangle.

$OD = DB + BO \Leftrightarrow R = 3r$.

The searched radius, expressed by the given radius R, is obtained by the formula:

$$r = \frac{R}{3}.$$

Problem 4: In a circle with a radius R eight identical circles $k_1(A,r), k_2(B,r), k_3(C,r), k_4(F,r), k_5(G,r), k_6(H,r), k_7(L,r), k_8(M,r)$ are entered, each is tangent to the other seven (**Figure 11**). Calculate the radius r.

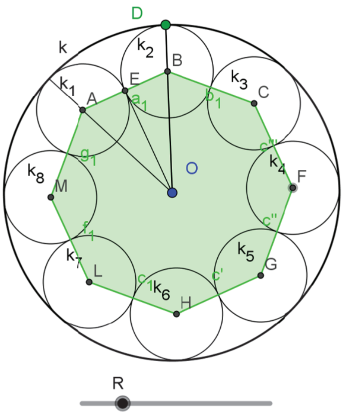

Figure 11

Solution: The points A, B, C, F, G, H, L, M are the vertices of the regular octagon $ABCFGHLM$.

$OD = R, BD = EB = r, OE \perp AB, AB = 2r$.

$OD = DB + BO \Leftrightarrow R = r + OB$.

From the formula of the radius of the circumscribed circle of a regular n-sided polygon, OB is obtained

$$OB = R = \frac{r}{\sin 22{,}5°},$$

from where the $OD = DB + BO \Leftrightarrow R = r + OB$ solution is obtained

$$R = r + \frac{r}{\sin 22{,}5°}.$$

The searched radius, expressed by the given radius R, is obtained by the formula:

$$r = \frac{R \sin 22{,}5°}{1 + \sin 22{,}5°}.$$

The newly created problem, Problem 4, suggests that it can be summarised as follows:

Problem 5 (Generalization): In a circle with a radius R n-number identical circles $k_1(A_1,r), k_2(A_2,r), k_3(A_3,r), \dots, k_n(A\_n,r)$ are entered, each is tangent to the great circle and the two of its identical circle. Calculate the radius r.

Solution: The points $A_1, A_2, A_3, \dots, A_n$ are the vertices of the regular n-sided polygon.
$OD = R, BD = EB = r, OE \perp AB, AB = 2r.$
$OD = DB + BO \Leftrightarrow R = r + OB$, where BO is the radius of the circumscribed circle of a regular n-sided polygon.

From the formula of the radius of the circumscribed circle of a regular n-sided polygon, OB is obtained

$$OB = R = \frac{r}{\sin\frac{180°}{n}},$$

from where for the equality $OD = DB + BO \Leftrightarrow R = r + OB$ is obtained

$$R = r + \frac{r}{\sin\frac{180°}{n}}.$$

The searched radius, expressed by the given radius R, is obtained by the formula:

$$r = \frac{R\sin\frac{180°}{n}}{1 + \sin\frac{180°}{n}}.$$

Problem 6 [5]: $k_1(A, r), k_2(B, r), k_3(C, r), k_4(F, r)$, each of which is tangent to the other two (**Figure 12**). Calculate the radius r, of the circle $k(O, R)$, which is tangent to all four circles, and is inscribed between them.

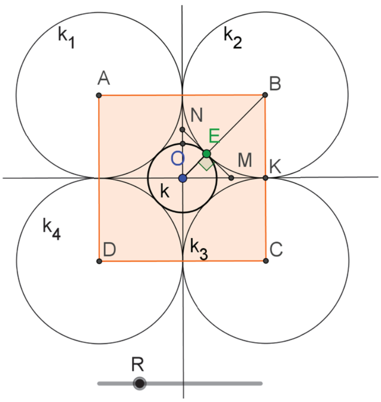

Figure 12

Solution: From the condition it follows that $OE = R$. Let us construct a segment EM, which touches the circle $k(O, R)$ and intersects the line OK at the point M. From where it follows that $EM \perp OE$.

Let us look at the triangles $\triangle OBK$ и $\triangle OME$:
1. $\sphericalangle BKO = \sphericalangle MEO = 90°$;
2. $\sphericalangle O$ – common angle,

therefore, the triangles are similar. From their similarity comes relationship between their sides:

$$\frac{OB}{OM} = \frac{BK}{EM}$$
$$\frac{r + R}{OM} = \frac{r}{EM} \qquad (1)$$

In the triangle $\triangle OEM$: $\sphericalangle OEM = 90°$, $\sphericalangle EOM = 45°$, then, $\sphericalangle OME = 45°$ and $OE = EM = R$, where for the segment OM the following is obtained:
$OM = \sqrt{R^2 + R^2} = R\sqrt{2}$.
In the triangle $\triangle OBK$: $OB = R + EB = R + BK = R + r$.
We substitute in the ratio (1) and we get:

$$\frac{r + R}{R\sqrt{2}} = \frac{r}{R} \Rightarrow r.R.R^2 = r.R\sqrt{2} \Rightarrow r.R\sqrt{2} - r.R = \sqrt{2} \Rightarrow$$

$$r\left(R\sqrt{2} - R\right) = R^2 \Rightarrow r = \frac{R}{R(\sqrt{2} - 1)}.$$

After the rationalizing of the denominator, the next step follows:

$$r = R\left(\sqrt{2} + 1\right).$$

CONCLUTIONS

The use of educational situations for the formation and development of mathematical problem posing and problem solving skills of the future teachers of mathematics based on the GeoGebra multi-platform foster the mathematical creativity of students. The proposed methodology of training contributes to the development of professional competence of future teachers of mathematics. It strengthens the interest of the future teachers to use GeoGebra in their mathematical educational activity.

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