Some learners' errors in teaching secondary school mathematics

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Abstract: Some learners' errors in teaching secondary school mathematics. *The paper analyses some typical and widely taken mistakes based mainly on false analogy and not understanding some basic mathematical concepts and ideas in teaching secondary school mathematics. The errors are systemized and some conclusions are made.*

Key words: *secondary school, teaching mathematics, typical error, false analogy, list of errors.*

INTRODUCTION

Teaching secondary school mathematics is a process that needs constant observation, analysis and conduct. When the everyday teaching takes place each participant in this process (teachers, pupils, parents, researchers, administrators, and management officers) indicate many cases of taking mistakes incidentally or more frequently. Some of these mistakes persist in time and cause difficulties and problems in future education or in real life. As a result of our long experience in this respect we try to explore some of these mistakes, the reasons for taking them and systemizing them as well. The aim is to achieve the "diagnose – treatment" effect i.e. if you know or recognize the problem, the reason for it you might more successfully find not only the way to overcome the problem but also to prevent pupils of future mistakes in teaching mathematics.

EXPOSITION

Students all too frequently make the substitution $\sqrt{a^2+b^2}=a+b$, where "=" designates "is incorrectly replaced by". Many common errors like this are shown to arise of two processes:

1. Inappropriate use of a known rule "as is" in a new situation.
2. Incorrect adaptation of a known rule to solve a new problem.

These processes are termed "reasonable" because often:

1. The rule that serves as the basic for extrapolation works correctly for problems that are nearly isomorphic variations of the prototype from which it is drawn.
2. The extrapolation techniques that specify ways to extend base rules are often useful techniques that apply correctly in other situations.
   The problem thus lies not in an extrapolation technique, but in the student's misguided belief in (or failure to evaluate) the appropriateness of using that technique in the particular situation at hand.

Consider, for example, the table below. Each of the entries marked "correct" – the collection of which constitutes a large part of a student's experience in mathematics – satisfies a "linear distribution law" of the form $f(x.y)=f(x).f(y)$. Each of the entries marked "incorrect" represents an unfortunate extrapolation of that law, generalized to $f(x*y)=f(x)*f(y)$, to a situation in which the extrapolation is not justified.

Similarly generalization enables the formulation on of a general rule from a sample problem based on the assumption that particular numbers in sample problems are incidental rather than essential. This assumption is nearly always valid, but there is a classic exception in the solution of problem like:

$(x-3)(x-4)=0 \Leftrightarrow x-3=0 \; or \; x-4=0 \Leftrightarrow x=3 \; or \; x=4$

Although the numbers 3 and 4 are not crucial to the procedure itself, the 0 is. Students who fail to realize the critical nature of the 0 treat it just as they do other numbers in the prototype and construct a rule of product solutions of the form:

$(x-5)(x-7)=3 \Leftrightarrow x-5=3 \; or \; x-7=3 \Leftrightarrow x=8 \; or \; x=10$.

It could be easily noticed that mistakes of this kind are taken because of:

- using false analogy and transferring the way of solution of one equation $((x-5)(x-7)=3)$ to another $((x-3)(x-4)=0)$ which is not analogous to the previous one;
- not understanding the equivalency $u.v=0 \Leftrightarrow u=0 \; or \; v=0$ and the role of the number 0 in it.

Learner's errors in teaching secondary school mathematics are a serious problem which is not amenable to easy solution. Because of the avoidance strategies which develop students who often do mistakes, learn little mathematics. What they do they learn or they memorize, they do it without understanding. It is to the teacher's advantage, then, to do everything possible to reduce the level of mathematics errors in the classroom. One of ways for reducing the number of student's mistakes is considering the table like done below. The other one is contributing to the class and comparing a lot of examples which illustrate correct and incorrect solution of problems.

CONCLUSION

Indicating, correcting and analyzing errors are the most important tasks in teaching mathematics. Each of the mistakes taken by the students should be considered as a signal to find the reason/reasons for doing so and eventually to search approaches to overcome it. This would provide better learning environment and more confidence in pupils when assimilating mathematical pieces of knowledge.

PROBLEMS FOR FURTHER RESEARCH

1. Regular registration and systemizing learners' mistakes in teaching secondary school mathematics.
2. Methods and organization forms for overcoming mistakes taken by pupils in teaching mathematics.
3. Examining some possibilities for more extracurricular activities as well as solving groups/systems of problems aiming at better understanding mathematical concepts and ideas.

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EXAMPLES OF CORRECT AND INCORRECT GENERALIZED DISTRIBUTION

correct	**incorrect**
$A.(B+C)=A.B+A.C$	$A.(B.C)=AB.AC$
$A.(B-C)=A.B-A.C$	$A.(B:C)=(A.B):(A.C)$
$\frac{1}{A}.(B+C)=\frac{1}{A}.B+\frac{1}{A}.C \Leftrightarrow \frac{B+C}{A}=\frac{B}{A}+\frac{C}{A}$	$\frac{1}{B+C}.A=\frac{1}{B}.A+\frac{1}{C}.B \Leftrightarrow \frac{A}{B+C}=\frac{A}{B}+\frac{A}{C}$
$\frac{A.C}{B.C}=\frac{A}{B}, C\neq 0$	$\frac{A+C}{B+C}=\frac{A}{B}$
$A.0=0$	$A+0=0$
$A.1=A$	$A+1=A$
$A+(-A)=0 \Leftrightarrow A-A=0$	$A.(-A)=0$
$A.\frac{1}{A}=1 \Leftrightarrow \frac{A}{A}=1$	$A+\frac{1}{A}=1$
$A^{m+n}=A^m.A^n$	$A^{m+n}=A^m+A^n$
$A^{m-n}=\frac{A^m}{A^n}$	$A^{m-n}=A^m-A^n$
$A^{m.n}=(A^m)^n$	$A^{m.n}=A^m.A^n$
$(A.B)^2=A^2.B^2$ more generally $(A.B)^n=A^n.B^n$	$(A+B)^2=A^2+B^2$ more generally $(A+B)^n=A^n+B^n$
$\sqrt{A.B}=\sqrt{A}.\sqrt{B}$ more generally $(A.B)^{\frac{1}{n}}=A^{\frac{1}{n}}.B^{\frac{1}{n}}$	$\sqrt{A+B}=\sqrt{A}+\sqrt{B}$ more generally $(A+B)^{\frac{1}{n}}=A^{\frac{1}{n}}+B^{\frac{1}{n}}$
$A.B=0 \Leftrightarrow A=0$ or $B=0$	$A.B=C, C\neq 0 \Leftrightarrow A=C$ or $B=C$
$\log_m(A.B)=\log_m A+\log_m B$	$\log_m(A+B)=\log_m A+\log_m B$
$\sin(A+B)=\sin A.\cos B+\cos A.\sin B$	$\sin(A+B)=\sin A+\sin B$