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INVESTIGATION OF ROBUST STABILITY OF ELECTRO-HYDRAULIC CONTROL MODULE FOR HYDRAULIC STEERING SYSTEM WITH LINEAR-QUADRATIC REGULATOR

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Abstract: *The paper investigates the robust stability of an embedded robust controller for optimal reference tracking of electrohydraulic steering systems. The regulator is sinthesized on the base of multivariable system identification and quadratic cost function. A Kalman filtering algorithm is used for the state estimation. In order to describe the system in robust control framework we introduce a small uncertain element into the model from identification in the form of input multiplicative uncertainty. Then the system is represented as a* $M-\Delta$ *interconnection which allows to calculate the structured singular value (*μ*) of the closed loop system with the linear quadratic regulator. This singular value is a measure of the loop stability in presence of bounded variations in the model characteristics in frequency domain or in its parameters. Therefore the present paper proves that the closed loop system keeps its stability in presence of unmodelled dynamic effects caused for example by the inherent nonlinearities in the hydraulic steering units.*

Keywords: *Linear-quadratic regulator (LQR), Kalman filter, Robust stability, Steering system.*

INTRODUCTION

The need for mobile machines with automated remote control is a determining factor for the development of the built-in electrohydraulic steering systems. A basic device in these systems is an electrohydraulic steering unit (EHSU). Modern EHSU enable reconciliation of two modes of steering depending on the control action: mechanical - through the steering wheel and digital - an electronic joystick or GPS signal. In this way, besides meeting the requirements of the safety standards, new advantages are gained in terms of precise remote control and providing a variable steering ratio between the steering wheel and the steered wheels based on dedicated control modules (Danfoss, 2016). An example of this is the well-known Danfoss PVE type of electrohydraulic control modules (EHCM). It has an electro-hydraulic system consisting of four two-way two-position valves of small size dimensionally connected in parallel, which serve for pilot hydraulic control of a proportional spool valve (with position feedback) which determines the direction of movement of the executive servo-cylinder. This requires the installation of an efficient embedded control system guaranteeing the quality of the entire electrohydraulic system.

The main purpose of the present work is to synthesize and implement a control device of the EHSU to ensure robust stability and quality of the control system. In order to achieve this goal, based on the identification model obtained, a linear-quadratic-Gaussian (LQG) regulator with integrated action is synthesized. Robust stability of the management system has been investigated using the developed model of EHSU with input multiplicative uncertainty. The synthesized regulator is implemented in a 32-bit microcontroller on a test bench for investigation of electro-hydraulic steering devices. A number of experiments have been carried out with the developed EHSU control system.

Experimental system layout

Experimental studies were performed on a laboratory test bench for electrohydraulic steering systems based on the OSPE 200 LSRM, taking into account the technical specifications of manufacturers of such systems and standards in their design - EU Machinery Directive 2006/42 / EC and ISO 13849-1 (Weber, J., 2016).

Fig. 1 shows the hydraulic scheme of the EHCM for the control of the EHSU in digital mode.

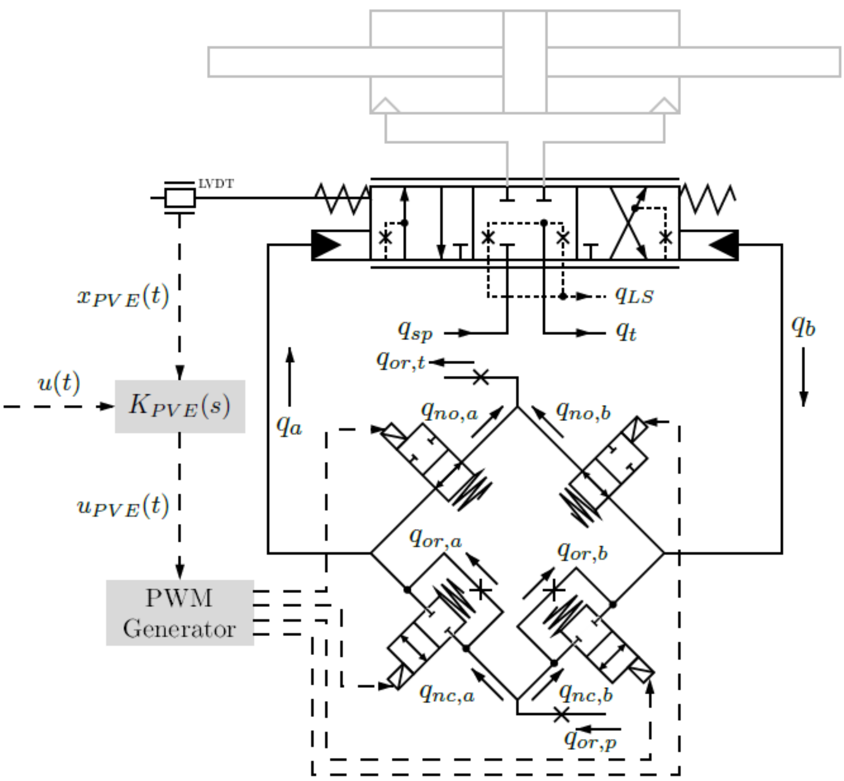

Fig. 1. Hydraulic diagram of EHCM

Design of LQG regulator for control EHCM

In order to design an LQG regulator there is necessary to have a linear state-space model of the plant. In this paper we use such model which is estimated with identification from experimental data (Mitov, Al., 2018). The model we have is

$$\begin{array}{c} x(k+1) = Ax(k) + Bu(k) + K_v v(k) \\ y(k) = Cx(k) + Du(k) + v(k) \end{array}, A = \begin{bmatrix} 0.8769 & -0.3987 & 0.3986 \\ 0 & 0 & 1 \\ -0.1666 & -0.5099 & 1.509 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.0043 \\ 0.0011 \\ 0.0021 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, K_v = \begin{bmatrix} 0.1112 & -0.06214 \\ -0.09525 & 1.55 \\ -0.2003 & 1.897 \end{bmatrix} \quad (1)$$

where $x(k) = [x_1 \quad x_2 \quad x_3 \quad]^T$ is a vector with the state variables, $u(k)$ is the input signal, $y(k) = [y_{pres} \quad y_{pos}]^T$ is the output signal, $v(k)$ is the residual error from the model, and $A, B, C, D. K_v$ are matrices with suitable dimensions. The firs output of the model (1) is the measured position of the piston, and the second output is the measured pressure drop across the cylinder chambers.

In order to achieve reference trajectory tracking we have designed the LQG regulator with included integral compoment (Goodwin, G., 2001). Therefore the determinicstic part of the model (1) is extended with the additional state x_i. This additional state is an ingegral of the position tracking error

$$x_i(k+1) = x_i(k) + T_s e_v(k) = x_i(k) + T_s(y_{ref}(k) - y_{pos}(k)), \quad (2)$$

where $y_{ref}(k)$ is the reference. By combining equations (1) and (2) there we have the following representationtion of the extended system.

$$\begin{array}{c} \bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}u(k) + \bar{G}y_{ref}(k), \\ y(k) = \bar{C}\bar{x}(k), \end{array}, \quad (3)$$

$$\bar{x}(k) = \begin{vmatrix} x(k) \\ x_i(k) \end{vmatrix}, \bar{A} = \begin{vmatrix} A & 0 \\ -T_s C & 1 \end{vmatrix}, \bar{B} = \begin{vmatrix} B \\ 0 \end{vmatrix}, \bar{C} = |C \quad 0|, \bar{G} = \begin{vmatrix} 0 \\ T_s \end{vmatrix} \quad (4)$$

The optimal control action is

$$u(k) = -\bar{K}\bar{x}(k), \bar{K} = [K_c \quad -K_i], \quad (5)$$

where K_c is a matrix of the proportional gains on each state and is K_i is the integral gain. The matrix of the regulator $\bar{K}$ then is calculated from

$$\bar{K} = (R + B^T P B)^{-1} B^T P A(k) = -\bar{K}\bar{x}(k), \bar{K} = [K_c \quad -K_i], \quad (6)$$

where P is the positive definite solution of the following Riccati equation

$$A^T P A - P - A^T P B(R + B^T P B)^{-1} B^T P A + Q = 0 \quad (7)$$

The optimal regulator is calculated for fixed values of the matrices

$$Q = \begin{bmatrix} \times 10^4 & 0 & 0 & 0 \\ 0 & 10^4 & 0 & 0 \\ 0 & 0 & 10^4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad R = 5000 \quad (8)$$

Since the state $x(t)$ of the plant (1) is not cannot be directly measured, the optimal control action (5) is implemented as

$$u(k) = -K_c\hat{x}(k) + K_i x_i(k), \quad (9)$$

where $\hat{x}(k)$ is the estimate of $x(t)$. It is calculated with a discrete Kalman filter

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K_f(y(k+1) - CBu(k) - CA\hat{x}(k)) \quad (10)$$

The matrix of the filter K_f is determined as

$$K_f = D_f C^T (CDC^T + 10^{-4} I_2)^{-1}, \quad (11)$$

where I_2 is a second order unit matrix and D_f is a positive definite solution of the Riccati equation

$$AD_fA^T - D_f - AD_fC^T(CDC^T + 10^{-4}I_2)^{-1}CD_fA^T + K_vD_vK_v^T = 0. \qquad (12)$$

The matrix $D_v = \begin{bmatrix} 108.97 & 0 \\ 0 & 27.44 \end{bmatrix}$ is a noise variance $v(k)$.

Robust stability of the designed LQG

The robustness of a system means that it retains certain properties, regardless of the variations in the parameters of its internal elements within the permissible limits. It is of interest to investigate whether the closed system with the linear quadratic regulator will retain its stability in the presence of variations in the parameters in the matrix B, which characterize the effect of the control signal on the object. This is due to the presence of an dead-band in the response of the main directional valve caused by the positive overlap of its edges which is designed in this way for safety considerations. This valve controls directly the direction of movement of the steering cylinder. There is assumed a 30% uncertainty in the matrix B,

$$\tilde{B} = B + \begin{pmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & 0 \\ 0 & 0 & \delta_3 \end{pmatrix} |0.3B|, \qquad (13)$$

where $\delta_1, \delta_2, \delta_3 \in [-1,1]$ are normalized unsertain scalar variables independent of each other. For certain specific values of the uncertain elements, a single implementation of the system is obtained. That's why the uncertain system becomes

$$\begin{cases} x(k+1) = Ax(k) + \tilde{B}u(k) \\ y(k) = Cx(k) \end{cases} \qquad (14)$$

is described by a family of characteristics corresponding to a different choice of values for uncertain variables. Fig. 6 present the transient responses in pressure drop and in position of the uncertain open system. As can be seen, the uncertainty leads to a variation in the static gain of the system. Fig. 7 shows the amplitude-frequency responses of the open loop for the pressure and position. The influence of the introduced uncertainty upon the pressure channel is significantly greater than upon the position channel. To investigate the effect of the uncertainty on the closed loop with the linear quadratic regulator we use the following expressions

$$\begin{cases} \begin{pmatrix} x(k+1) \\ x_{LQG}(k+1) \end{pmatrix} = \begin{pmatrix} A & \tilde{B}C_{LQG} \\ -B_{LQG}C & A_{LQG} \end{pmatrix} \begin{pmatrix} x(k) \\ x_{LQG}(k) \end{pmatrix} + B_{LQG} \begin{pmatrix} 0 \\ r(k) \end{pmatrix} \\ y(k) = (C \quad 0) \begin{pmatrix} x(k) \\ x_{LQG}(k) \end{pmatrix} \end{cases}, \qquad (15)$$

where $A_{LQG}, B_{LQG}, C_{LQG}$ is the state-space representation of the designed linear-quadratic regulator and x_{LQG} is its internal state vector. There can be observed that the uncertain elements from the open-loop matrix $\tilde{B}$ are expressed also in the matrix A_c of the closed loop. Hence the introduced uncertainty can influence the stability of the loop.

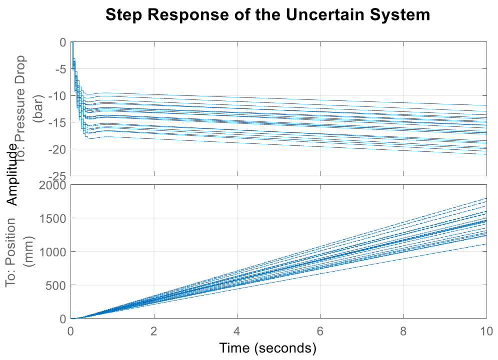

Fig. 6. Step response of uncertainty open loop system

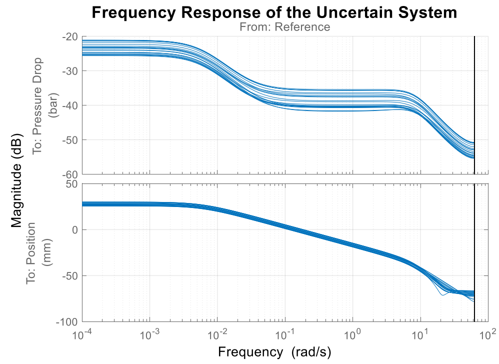

Fig. 7. Frequency response

Fig. 8 shows the family of transient response of the closed loop in position, and Fig. 9 shows the amplitude-frequency response of the closed system with respect to the signal r. The important thing here is that the closed-loop system retains its performance despite the presence of parametric disturbances. To investigate the robust stability of the closed loop system we use the following representation

$$\begin{cases} \begin{pmatrix} x(k+1) \\ x_{LQG}(k+1) \end{pmatrix} = \begin{pmatrix} A & BC_{LQG} \\ -B_{LQG}C & A_{LQG} \end{pmatrix} \begin{pmatrix} x(k) \\ x_{LQG}(k) \end{pmatrix} + \begin{pmatrix} 0 & \mathrm{I} \\ 0 & 0 \end{pmatrix} u_\Delta \\ y_\Delta = \begin{pmatrix} 0 & 0.3|B|C_{LQG} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x(k) \\ x_{LQG}(k) \end{pmatrix} \end{cases}, \tag{16}$$

where $u_\Delta = \Delta\, y_\Delta$ and $\Delta = diag(\delta_1, \delta_2, \delta_3)$. This representation is widely known as $M-\Delta$ structure in robust control theory (Petkov, P., 2018) and decompose the uncertain system to a nominal deterministic part and an uncertain matrix. In order to characterize the robust stability of the system there is defined the structured singular value μ as a reciprocal of the minimal as norm uncertainty Δ, which would make the system internally unstable

$$\mu_\Delta(M) = \sup_\omega \frac{1}{\min\{\|\Delta\|_\infty | \det(j\omega I - A_c(\Delta)) = 0\}} \tag{17}$$

The robust stability of the closed loop system for the entire range of uncertain perturbatuions i.e. $\|\Delta\|_\infty \leq 1$ is equivalent to $\mu_\Delta(M) < 1$. From Fig. 10, we can determined an upper bound for the $\frac{1}{\min\{\|\Delta\|_\infty | \det(j\omega I - A_c(\Delta)) = 0\}}$. It can be seen that the upper bound of the structured singular number is about 0.3, which confirms that the closed uncertain system is robustly stable to the uncertainty in the matrix B of 30%. It can also be said that the system will maintain its robust stability even if this uncertainty is increased twice.

Experimental results

The designed linear-quadratic regulator is embedded into microcontroller MC012-022. The regulator is represented in the following vector matrix form. Thus, the calculation of the control action is reprsented as multiplying a matrix by vector from a computational viewpoint.

$$\begin{pmatrix} \hat{x}(k+1) \\ x_i(k+1) \\ u(k+1) \end{pmatrix} = \begin{pmatrix} A - CA & 0 & B - CB \\ -T_S C & 1 & 0 \\ K_c & -K_i & 0 \end{pmatrix} \begin{pmatrix} \hat{x}(k) \\ x_i(k) \\ y(k) \end{pmatrix} \tag{18}$$

$$y(k) = \begin{pmatrix} y_{ref}(k) & y_{press}(k) & y_{pos}(k) \end{pmatrix}^T$$

In this microcontroller, the visual programming approach can be used, where the algorithm is introduced as a block diagram through standard functional blocks. The matrix is represented as a one-dimensional array by row concatenation and indexed from a dedicated counter. The matrix elements are represented as numbers in a fixed point format, scaled by a factor of 1000. Fig. 11

shows the response of the piston of the cylinder when reference trajectory is periodic rectangular signal in both directions relative to the middle position. This is a typical signal for low-speed machines. During the transients, short-time stops are observed in several intermediate positions. The reason for this is the presence of a dead-band nonlinearity in the proportional valve built into the EHSU due to its construction features. The error in steady state is close to zero. The quality of the transients is maintained in both directions.

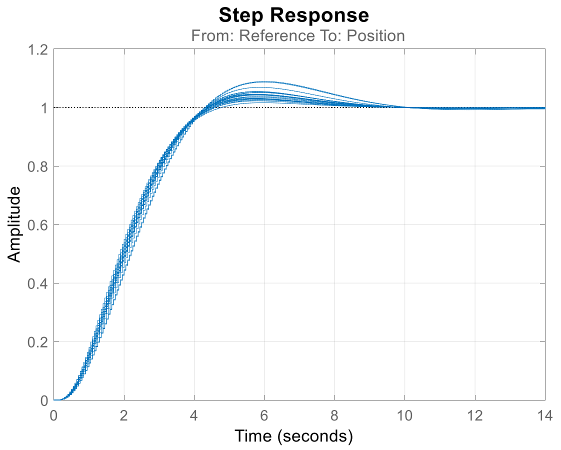

Fig. 8. Step response of the closed loop position system.

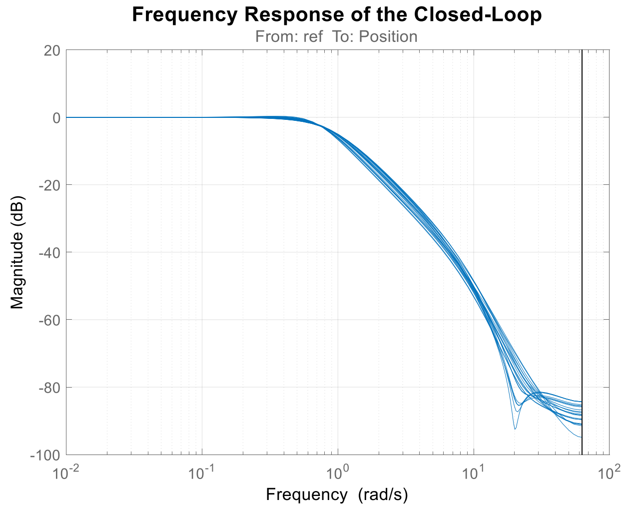

Fig. 9. Frequency response of the closed loop position system.

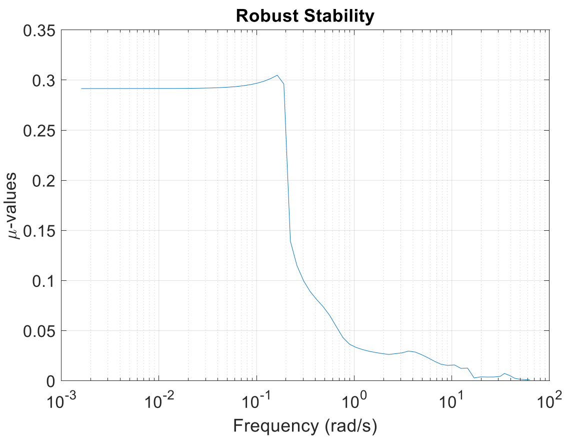

Fig. 10. Robust stability of uncertainty closed loop system

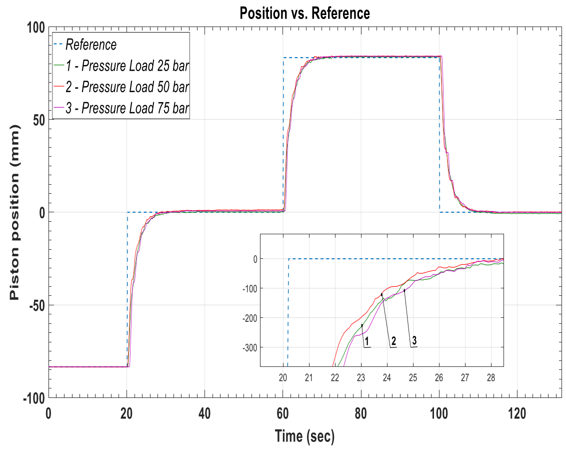

Fig. 11. Experimental transient response of the closed loop system

CONCLUSION

The article presents the developed control algorithm of the EHCM, which provides robust stability and performance. The system is based on a synthesized LQG regulator with integral action. Robust stability was investigated using the identified model of EHCM with input multiplicative uncertainty, which takes into account the deviations of the parameters that characterize the way the control signal acts on the state of the model. A number of experiments with the developed regulator have been executed, which confirm the quality of the EHCM control system.

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