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APPLICATION OF THE PEAKS-OVER-THRESHOLD METHOD ON ANALYSIS OF GOLD PRICE

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Abstract: *Financial markets and financial instruments have experienced many changes during the last decades. But despite these developments, the gold has begun to regain its historical significance in recent years. This study includes a brief introduction to the importance of gold for the global financial system and to the method of surpluses exceeding the maximum threshold, as well as its application in the analysis of the market risk for gold price. A forecast assessment of the Value at Risk and the Expected Shortfalls for the period of years from 2021 to 2030 has been made. An ordinary differential equation and cftool in Matlab were also applied in calculations of the predictive estimates of* VaR *and* ES.

Keywords: *Gold Model, Threshold Method, Risk Value, Expected Shortfall, New Bretton Woods, Financial Market*

INTRODUCTION

Peaks over threshold method (POT) is one of the main approaches for practical extreme value analysis. It is based on the consideration of the extreme values of a statistical sample, which exceed a previously defined threshold value. The present study begins with a description of the importance of gold for the global financial system and continues with an introduction to peaks over threshold method, as well as its application in the analysis of market risk for the price of gold, using a statistical sample of historical data from 1971 to 2020. A predictive estimates of the Value at Risk $\left(\widehat{VaR_p}\right)$ and the Expected Shortfall $\left(\widehat{ES}_p\right)$ has been calculated for the period from 2021 to 2030 with safety factor of $p = 1\%$. Ordinary differential equation is used to calculate the predictive estimates of $\widehat{VaR}_p$ and $\widehat{ES}_p$. Gold is one of the most valuable natural resources due to its influence and importance for trade and the stability of the global financial and economic system. In 2020, gold began to regain its historical significance and role, which could be the key to opening new horizons and opportunities for human civilization.

EXPOSITION

The history of gold has long been associated with money, but after World War II, gold gradually lost its role in developed economies. At the end of the war, a monetary system, which is known as the Bretton Woods system was created. This system lasted until 1971, when the United States unilaterally abolished its gold standard, which had set the convertibility of gold with the dollar at USD 35.00 per troy ounce. The Bretton Woods system is an international system for the organization of trade payments and exchange rates, established as a result of a special international conference from 01 to 22 July 1944 in Bretton Woods, USA. At this conference it was decided to organize the world monetary system around the US dollar, which in turn will be linked to a fixed price of gold. Thus, the US dollar became a global currency. In November 1961, the London Gold Pool was formed, which brings together the gold reserves of eight US banks and the central banks of seven European countries to keep the price of the precious metal at USD 35 an ounce and prevent the price of gold from rising. However, as the Vietnam War has deepened US budget spending increased and the world was flooded with increasingly depreciating paper dollars. In March 1968, a two-tier gold market was introduced with a free-floating private market and official fixed parity transactions. The two-tier system is fragile in nature. The US deficit problem has continued and intensified. As speculation against the dollar intensified, other central banks became increasingly reluctant to accept dollars in settlement. The situation became unbearable, and finally on August 15, 1971, President Nixon announced that the United States would stop converting the dollar on demand into gold for the central banks of other countries.

After the end of the Bretton Woods system, gold began to be traded freely on world markets and any kind of gold standard became unfeasible. A few years later, after the oil crisis in 1973, the world economy shifted to a new system, later called the Petrodollar system. It has turned the dollar into a global reserve currency, and through this status the United States enjoys a constant trade deficit and global economic hegemony to nowadays. After 2020, the petrodollar fiat system is facing its biggest challenge to date, and is partly dragging the global economy into a deep economic crisis. Some economists call it the "Crisis of the Century" and/or state that we need a New Bretton Woods.

Based on the recent global economic past and the importance of gold for the world economy and world markets, this publication will provide a mathematical analysis of the behavior of gold and the risk of a sharp rise in price. Estimates of the Value at Risk and the Expected Shortfall will be made. For the purpose of the analysis, the peaks over threshold method will be used, which is one of the two approaches for practical analysis of extreme values. This analysis is used as a tool to analyze and study the values of the sample, which deviate exclusively from the average value of the complete sample. The method of peaks over threshold is based on the consideration of extreme values of a statistical sample. The basic concept of the method is to use a threshold to isolate values considered finite from the rest of the data, and to create a model for the finite values by modeling the tail of all values exceeding this threshold. In practice this is done by setting a threshold $u -$ a value defined in $\mathbb{R}$, which exceeds most but not all values, defined in some time series or some other vector of values (Franke, Härdle & Hafner (2008)).

Assessing the probability of rare and extreme events is an important issue in financial risk management. Extreme values theory provides the necessary basis for statistical modeling of such events, and of extreme risk measures. Some of the most common issues related to financial risk management are related to the assessment of extreme quantiles. This corresponds to determining the value that a given variable exceeds with a given probability. Value at risk is an example of such a measure (VaR).

Value at Risk is defined as the capital sufficient to cover the losses of the portfolio for a certain period of time. We assume that the random variable X with a continuous distribution function F models the losses of a certain financial instrument for a certain period of time. The value VaR_p could be defined as the p-th quantile of the distribution function F.

$$VaR_p = F^{-1}(1-p), \tag{1}$$

Here F^{-1} is the so-called function of quantiles, which is inverse of the distribution function F.

Expected Shortfall (ES) is another informative measure of the risk or contingent expectation that estimates the potential amount of loss in excess of VaR:

$$ES_p = E\left(X | X > VaR_p\right) \tag{2}$$

In other words, the Expected Shortfall is a risk concept used in the field of measuring financial risk to assess market risk or credit risk of the portfolio. Expected Shortfall is an alternative to the Value at Risk, which is more sensitive to the shape of the tail. Expected Shortfall is also

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called conditional value at risk (CVaR), average value at risk (AVaR), expected tail loss (ETL) and superquantile. Shortfall is considered to be more useful in measuring the risk than VaR.

Marginal distribution of observations exceeding the maximum threshold

The maximum threshold method takes into account the distribution of values exceeding a defined threshold. Figure 1 shows an (unknown) function of distribution F of a random variable X. The object of interest is the evaluation of the distribution function F_u of the values of x, exceeding a certain threshold u. The distribution function F_u is called a conditional distribution function of values exceeding the maximum threshold and is defined as:

$$F_u(y) = P(X - u < y | X > u), 0 < y < x_F - u ,$$

where X is a random variable, u is a threshold, $y = x - u$ is equal to the value of X exceeding the threshold u and $x_F \leq \infty$ is the rightmost endpoint of F. F_u could be written using F in the following way.

$$F_u(y) = \frac{\left(F(u + y) - F(u)\right)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)} \quad (3)$$

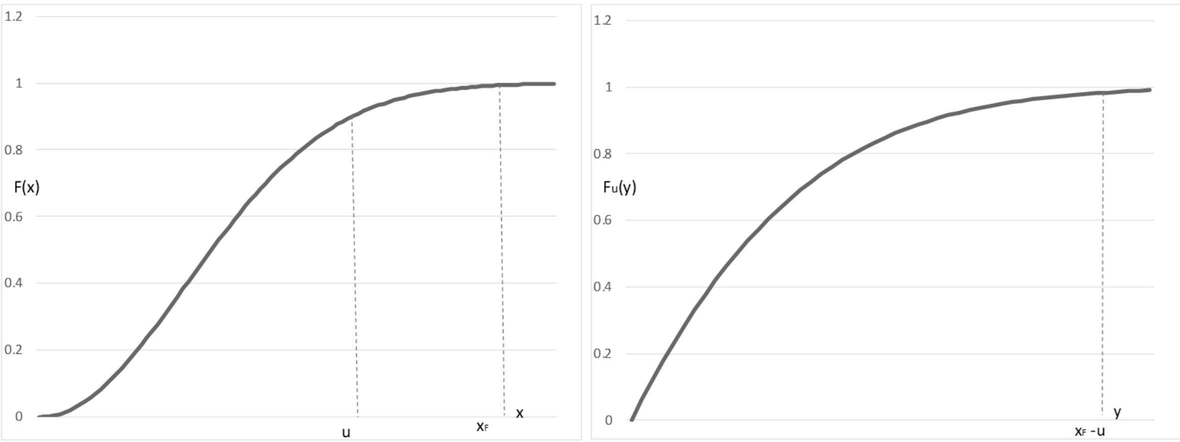

Figure 1. Distribution function $F(x)$ and conditional distribution function $F_u(y)$

The values of the random variable X are mostly located between 0 and u and therefore the estimate of F in this interval usually is not a problem. However, estimating F_u can be difficult, as there are usually very few observations in this area. The extreme value theory gives a powerful result for the conditional distribution function of values exceeding a sufficiently high (maximum) threshold, which is given in the following theorem:

Theorem 1. (Pickands (1975), Balkema and de Haan (1974)). For a large class of distribution functions F the conditional distribution function F_u на of peaks over values exceeding the maximum threshold, for large u is well approximated by the following function:

$$F_u(y) \approx G_{\xi,\sigma}(y), \qquad u \to \infty$$

where

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi}{\sigma} y\right)^{-\frac{1}{\xi}}, when\ \xi \neq 0 \\ 1 - e^{-y/\sigma} \qquad, when\ \xi = 0 \end{cases} \quad (4)$$

for $y \in [0, (x_F - u)]$, when $\xi \geq 0$ and $y \in [0, -\sigma/\xi]$, when $\xi < 0$. $G_{\xi,\sigma}(y)$ is called generalized Pareto distribution (GPD) .

If x is defined as $x = u + y$, then the generalized Pareto distribution can be expressed as a function of x as follows:

$$G_{\xi,\sigma}(x) = 1 - \left(1 + \frac{\xi(x-u)}{\sigma}\right)_{+}^{-\frac{1}{\xi}}$$

The parameter of the tail ξ gives an indication of the tail weight, the larger ξ the heavier the tail.

- At $\xi > 0$, long-tail distribution is obtained – standard Pareto distribution.
- At $\xi \to 0$, exponential distribution with an average value σ is obtained.
- At $\xi < 0$, distribution with an extreme boundary point on the right is obtained $-\sigma/\varepsilon$.

As there can generally be no upper limit for financial losses, only distributions with values of the parameter $\xi \geq 0$ are suitable for modeling the distributions of returns of financial instruments. If we accept that the generalized Pareto distribution is the distribution of the tail, then we can obtain analytical expressions for VaR_p and ES_p as functions of the parameters of generalized Pareto distribution. Expressing $F(x)$ from (3) we obtain:

$$F(x) = \big(1 - F(u)\big)F_u(y) + F(u)$$

After substitution of F_u with the generalized Pareto function and $F(u)$ with $\left(\frac{n-N_u}{n}\right)$, where n is the total number of observations and N_u is the number of observations over the threshold u, we obtain:

$$\hat{F}(x) = \frac{N_u}{n}\left(1 - \left(1 + \frac{\hat{\xi}}{\hat{\sigma}}(x-u)\right)^{-\frac{1}{\hat{\xi}}}\right) + \left(1 - \frac{N_u}{n}\right),$$

which could be simplified to

$$\hat{F}(x) = 1 - \frac{N_u}{n}\left(1 + \frac{\hat{\xi}}{\hat{\sigma}}(x-u)\right)^{-\frac{1}{\hat{\xi}}} \tag{5}$$

Then from equations (1) and (5) for a given probability p is obtained:

$$\widehat{VaR}_p = u + \frac{\hat{\sigma}}{\hat{\xi}}\left(\left(\frac{n}{N_u}p\right)^{-\xi} - 1\right) \tag{6}$$

Equation (2) for expected shortfall could be written as:

$$\widehat{ES}p = \widehat{VaR}_p + E\big(X - \widehat{VaR}_p \big| X > \widehat{VaR}_p\big)$$

where the second term on the right is the expected value of the excess over the threshold $\widehat{VaR}_p$. It is known that the mean excess value for generalized Pareto distribution with parameter $\xi < 1$ has the form:

$$\mathrm{e}(z) = E(X - z|X > z) = \frac{\sigma + \xi z}{1 - \xi}, \sigma + \xi z > 0 \tag{7}$$

This function gives the mean excess value of X at various values of the threshold z. Another important result for the existence of moments is that if X has a generalized Pareto distribution, then for all natural numbers r, such that $r < \frac{1}{\xi}$, the first r moments will exist.

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Using Equation (2) for Expected Shortfall and the equation (7) above, by substituting $z = \widehat{VaR}_p - u$, and when X represents the excess of y over u, then we obtain:

$$\widehat{ES}_p = \widehat{VaR}_p + \frac{\hat{\sigma} + \hat{\xi}\left(\widehat{VaR}_p - u\right)}{1 - \hat{\xi}} = \frac{\widehat{VaR}_p}{1 - \hat{\xi}} + \frac{\hat{\sigma} - \hat{\xi}u}{1 - \hat{\xi}} \tag{8}$$

Threshold defining for peak values selection

For the purpose of the analysis, a sufficiently high threshold is defined and observations that exceed this threshold are selected. The threshold define is subject to a compromise between the variability and the bias of the assessment. If the threshold is lower, then as the number of observations increases, observations close to the center of the distribution function may be included in the series of maxima. Subsequently, the tail index is more accurate (less volatile), but displaced. On the other hand, choosing a high threshold reduces the bias, but makes the assessment more variable (fewer observations). A problem with the independence of observations may occur. The mean excess of the generalized Pareto distribution is a linear function (tends to infinity). According to Theorem 1, for a high threshold the series of the peaks over threshold approaches the generalized Pareto distribution. It is possible to choose the threshold at which convergence with the generalized Pareto distribution is achieved by finding an area with a linear shape of the graph.

The assessment of the generalized Pareto distribution includes two steps:

1. Choosing the threshold u. Mean excess plot could be used, when u is defined, such that $e(x)$ is an approximately linear function for $x > u$ ($e(u)$ is linear for the Generalized Pareto distribution).
2. The estimates of the parameters ξ and σ can be performed using the maximum likelihood method. After assessing the peaks over threshold distribution, the p-quantile estimate can be used in estimation of the extreme VaR.

The estimate of the $p -$ quantile is given by the formula:

$$\hat{x}_p = u + \frac{\hat{\sigma}}{\hat{\xi}}\left(\left(\frac{n}{N_u}p\right)^{-\hat{\xi}} - 1\right)$$

Mean excess function

Definition: Let X be a random variable and x_F upper bound of the values of X, then:

$$\mathrm{e}(u) = E(X - u|X > u), 0 \leq u < x_F$$

is called Mean excess function, $\mathrm{e}(u)$ gives the mean value above the threshold u.

If X is exponentially distributed random variable with parameter λ, then $e(u) = \lambda^{-1}$ for each $u > 0$. For generalized Pareto distribution we have:

$$\mathrm{e}(u) = \frac{\sigma + \xi u}{1 - \xi}, \sigma + \xi u > 0.$$

A graphical test of tail behavior can be performed based on the shape of the mean excess distribution. Let $X_1, X_2, \cdots, X_n$ be independent uniformly distributed random variables with an empirical distribution function F_n and let $\Delta_n(u) = \{i, i = 1, \cdots, n, X_i > u\}$. Then

$$e_n(u) = \frac{1}{count\left(\Delta_n(u)\right)} \sum_{i \in \Delta_n(u)} (X_i - u), u \geq 0,$$

where $count$ gives us the number of points in the series $\Delta_n(u)$.

The set Множеството $\{X_{k,n}, e_n(X_{k,n}), k = 1, \cdots, n\}$ gives us the mean excess plot. In heavy-tailed distributions, the function $e(u)$ tends to infinity for a high threshold value u (linear curve with positive slope).

Market Risk Assessment of Gold Price (XAU)

In order to assess the market risk for gold, the period and characteristics of the historical data that will be used for analysis must first be carefully selected. In this regard, we will go through the following two steps:

- Selection of the historical period of the data used for the analysis;
- Comparison of daily, weekly and monthly data. Analysis of their features and behavioral nature. Selection of the most suitable option for the analysis.

The history of finance after World War II could be divided into two main periods. The first one is the period of the partial gold standard set in Bretton Woods, and the second one is from the end Bretton Woods to the present days, known with the fiat monetary standard, closely related to the price of oil. The Bretton Woods system determined the direction of world finances until 1971, when the United States unilaterally terminated the convertibility of the USD with gold, which leaded to the end of the Bretton Woods system and turned the dollar into a fiat currency.

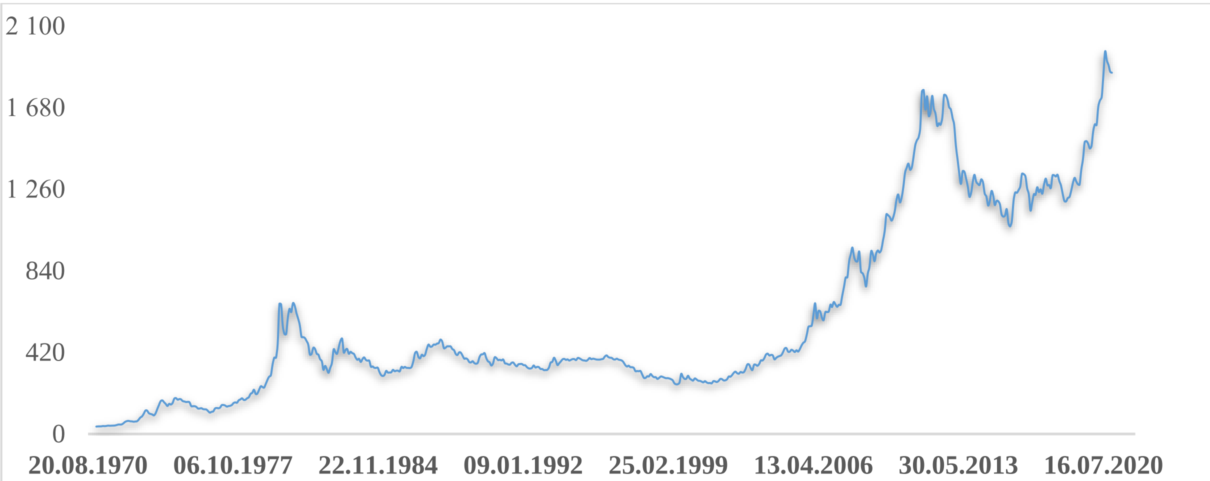

Figure 2. Price per troy ounce of Gold in USD for the period from 1971 to 2020

After August 15, 1971, gold prices began to rise gradually, but the US dollar retained its strength as a reserve currency, becoming the main currency in petroleum products trading. Therefore, the world financial system after 15.08.1971 is called the Petrodollar system. Although many financiers and economists have written about the shortcomings of the fiat monetary system in their studies, hardly in 2020, after the onset of the economic turmoil, the discussions of a new global financial system mainly came out. Therefore, for the purpose of the analysis, the gold values for the period from 1971 to 2020 will be used. When preparing for the analysis, one of the most difficult choices is what data to be used, whether daily, weekly or monthly. For this purpose, we will analyze the behavior, volatility, jumps and declines of the gold price.

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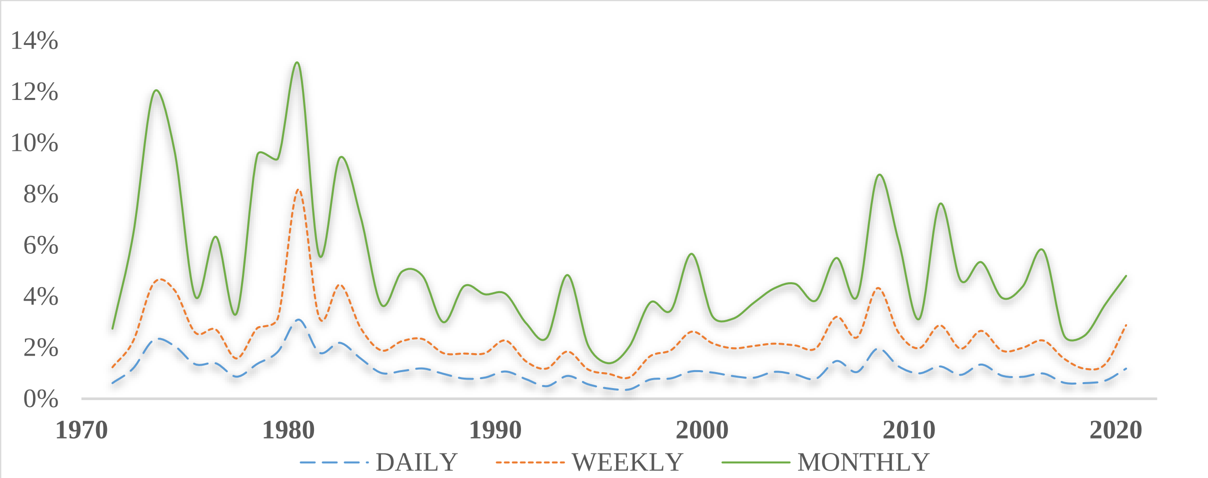


Figure 3. Volatility of Gold price from 1971 to 2020.

Figure 3 shows that the volatility of monthly returns is higher than both the volatility of weekly returns and the volatility of daily returns. Based on this fact, it could be concluded that the significant change of gold price is a smooth process that occurs over a longer period of time. Significant changes in the value of gold do not occur within a few days, but gradually as a medium-term or long-term trend.

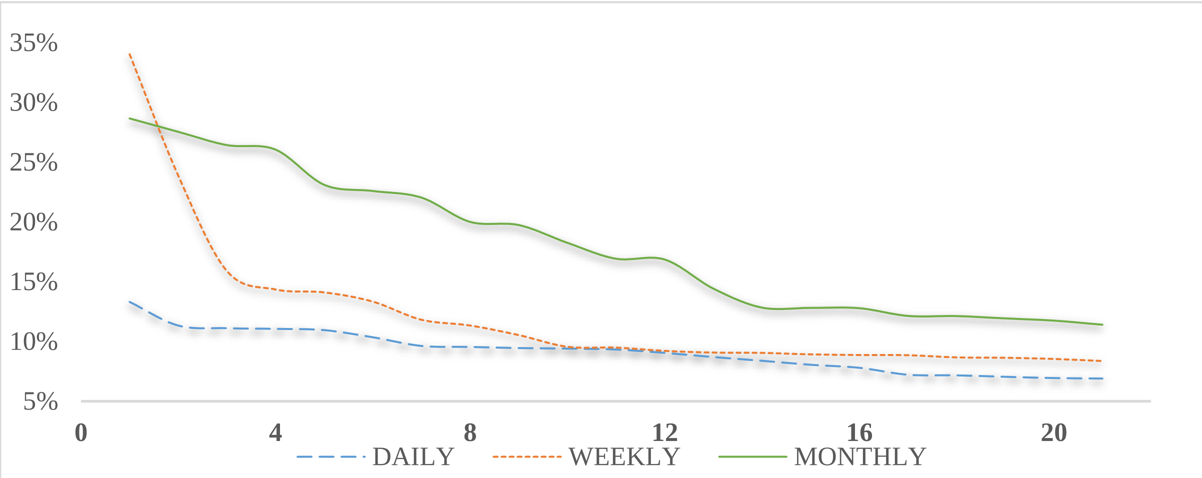


Figure 4. Largest monthly, weekly and daily jumps of Gold price

Proof of this could be seen in Figures 4 and 5. Figure 4 presents the largest jumps in the price of gold, where price changes are reported daily, weekly and monthly. It could be seen that the main part of the jumps reported as monthly changes are larger than those reported as daily or weekly changes. A similar statement can be made for the declines shown in Figure 5, where could be seen that the declines reported as monthly changes are larger than those reported as daily or weekly changes. This could be accepted as a second proof of the above-mentioned statement. Therefore, for market risk assessment of gold (XAU) will be used **monthly values** for the period from January 1, 1971 to December 31, 2020.

Before assessing the market risk for troy ounce of gold (XAU), referring to the Law on Large Numbers, it should be noted that the accuracy of the assessment directly depends on the number of observations and as the number of observations increases, the accuracy of the assessment also improves. Therefore, the market risk assessment will be improved by the accumulation of market observations. Accordingly, the assessment of the market data made with the observations until 31.12.2020 would not be completely accurate for the future periods. The following algorithm will be applied to solve this problem:

The historical data that is used in analysis and covers the time range from 1 January 1971 to 31 December 2020 is equal to 50 years. Firstly we will divide this range into five periods and make a forecast of the sixth future period as follows:

Period I - from January 1, 1971 to December 31, 1980
Period II - from January 1, 1981 to December 31, 1990
Period III - from January 1, 1991 to December 31, 2000
Period IV - from January 1, 2001 to December 31, 2010
Period V - from January 1, 2011 to December 31, 2020
Period VI - from January 1, 2021 to December 31, 2030 (forecasted)

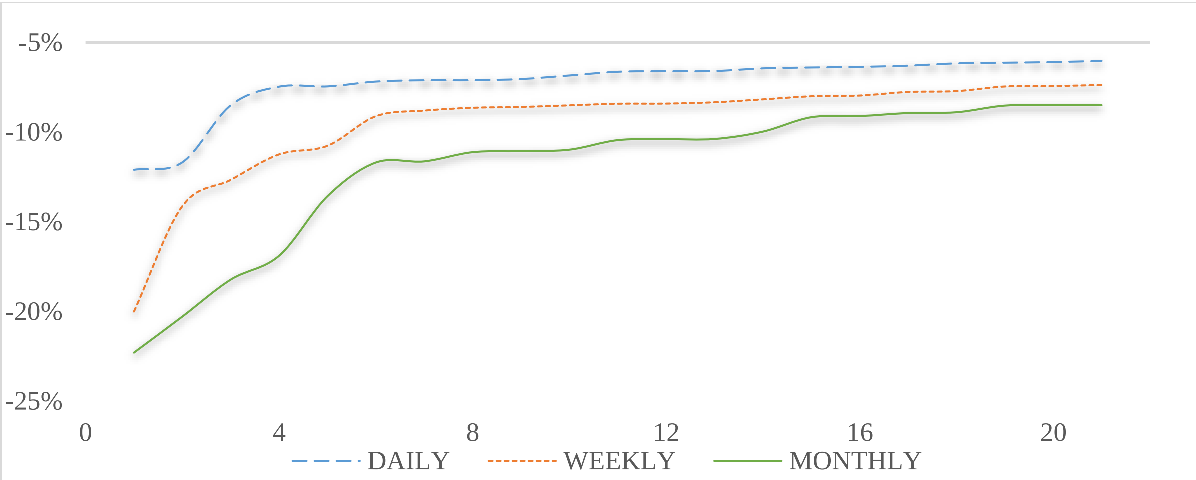


Figure 5. Largest monthly, weekly and daily declines in Gold price

Using the data from each of the above periods, the analysis of peaks over threshold will be applied by going through the following steps:

- Defining the optimal values of thresholds u for the periods from I to V.
- Defining the optimal parameters of the distribution function of peaks for the periods from I to V.
- Calculation of VaR_p и ES_p values for the periods from I to V.
- Forecasting the parameters of the distribution function of peaks and the size of the maximum threshold u for period VI.
- Calculating of VaR_p and ES_p values for the period VI.

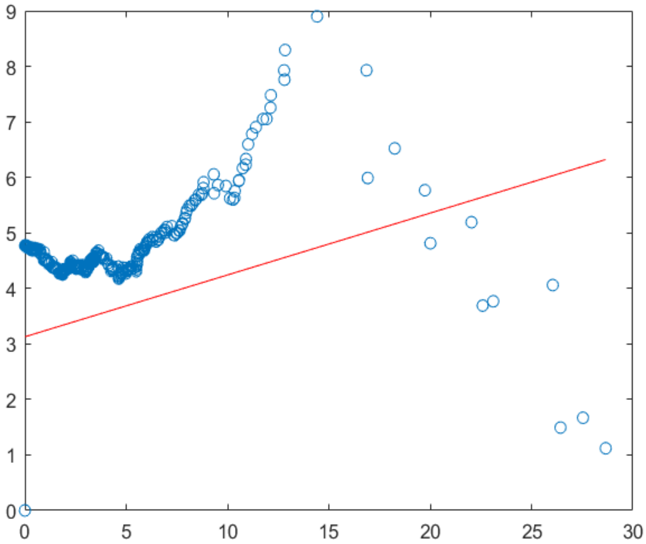
In addition, will be experimentally proven that the estimates of VaR_p and ES_p are not constant, change over time and with occurrence of new observations.

Peaks over threshold analysis

Selection of the maximum threshold value

The threshold u must be large enough to satisfy the conditions of Theorem 1 and at the same time it should not be too big, so that sufficient observations can remain in the sample. A graphical tool that is very useful when choosing a threshold is the mean excess plot defined by the points:

$$\left(u, e_n(u)\right), x_1^n < u < x_n^n$$



Where $e_n(u)$ are the mean excess of the sample:

$$e_n(u) = \frac{\sum_{i=1}^{n}(x_i - u)}{n - k + 1}, k = min\{i|x_i^n > u\}$$

and $n - k + 1$ is the number of observations over the threshold u.

The mean excess plot should be linear. This feature could be used as a selection criterion for u.

Figure 6 shows mean excess plot for the positive returns of data per troy ounce of gold for period **V**.

For the threshold of the right tail is obtained $u = 6.31$. This value is located there where the function becomes approximately linear.

Figure 6. Mean excess plot

The maximum likelihood method will be used to analyze the parameters over the periods. The maximum likelihood function on which the maxima should be found is as follows:

$$L(\xi, \sigma|y) = \begin{cases} -nln\sigma - \left(\frac{1}{\xi} + 1\right)\sum_{i=1}^{n} ln\left(1 + \frac{\xi}{\sigma}y_i\right), \text{when } \xi \neq 0 \\ -nln\sigma - \frac{1}{\sigma}\sum_{i=1}^{n} y_i, \text{ when } \xi = 0 \end{cases}$$

For $\xi \neq 0$ the maximum likelihood estimates of σ and ξ are obtained from the equation:

$$\frac{\partial L(\xi, \sigma|y)}{\partial \sigma} = 0, \leftrightarrow g(\xi, \sigma) = -\frac{n}{\sigma} + \left(\frac{1}{\xi} + 1\right)\sum_{i=1}^{n} \frac{\xi y_i}{\left(1 + \frac{\xi}{\sigma}y_i\right)\sigma^2} = 0$$

$$\frac{\partial L(\xi, \sigma|y)}{\partial \xi} = 0, \leftrightarrow f(\xi, \sigma) = \frac{1}{\xi^2}\sum_{i=1}^{n} ln\left(1 + \frac{\xi}{\sigma}y_i\right) - \left(\frac{1}{\xi} + 1\right)\sum_{i=1}^{n} \frac{y_i}{\left(1 + \frac{\xi}{\sigma}y_i\right)\sigma} = 0$$

For Pareto Type I distribution, the distribution function has the form:

$$F(x, \alpha) = 1 - \left(\frac{u}{x}\right)^{\alpha} \text{ за } x \geq u,$$

the distribution density will have the form:

$$f(x, \alpha) = \frac{\alpha u^{\alpha}}{x^{\alpha+1}} \text{ за } x \geq u,$$

Then the maximum likelihood function on which the maxima should be found is as follows:

$$L(\alpha|x) = \sum_{i=1}^{n} \ln\left(\frac{\alpha u^{\alpha}}{x_i^{\alpha+1}}\right) = nln\alpha + n\alpha lnu - \sum_{i=1}^{n}(\alpha + 1)lnx_i$$

Then the maximum likelihood estimate for α is obtained from the equation:

$$\frac{\partial L(\alpha|x)}{\partial \alpha} = 0, \leftrightarrow f(\alpha) = \frac{n}{\alpha} + nlnu - \sum_{i=1}^{n} lnx_i = 0$$

Whence the maximum likelihood estimate for α will be:

$$\hat{\alpha} = \frac{n}{(-nlnu + \sum_{i=1}^{n} lnx_i)}$$

The analysis of the first period includes 120 observations (10 years $\times$ 12 months), in calculations for each subsequent period we will use the previous observations too, ie it will include 120 observations more than the previous one. The number of observations over the threshold u will be N, where $N = n - k + 1$.

The value of the threshold is denoted by u. After applying the formulas (6) for $\widehat{VaR}_{0.01}$ and (8) for $\widehat{ES}_{0.01}$ and $\xi = \alpha^{-1}$, we obtain the estimates for $\widehat{VaR}_{0.01}$ and $\widehat{ES}_{0.01}$.

$$\hat{\sigma} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left|x_i - \left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)\right|^2}$$

The results of the analysis are shown by periods in Table 1.

Table 1. Peaks over threshold analysis of troy ounce gold values

Period	I	II	III	IV	V
u	9.7500	8.2800	7.7600	6.7600	6.3100
N	16	26	34	52	73
$\hat{\alpha}$	1.7256	1.7831	1.9674	1.9463	2.0909
$\hat{\sigma}$	6.7773	6.7549	6.6987	6.0653	5.5656
$\hat{\xi}$	0.5795	0.5608	0.5083	0.5138	0.4783
$\widehat{VaR}_{0.01}$	50.5243	42.0608	35.8426	35.1084	33.1187
$\widehat{ES}_{0.01}$	122.8326	100.5778	78.4929	77.5406	68.3622

Time series analysis (TSA) is very popular for modeling and analyzing market data time series. But one of the disadvantages of this method is that a large amount of market data is required. In such cases, numerical methods (NM), ordinary differential equations (ODE), partial differential equations (PDE) or stochastic differential equations (SDE) are considered and applied. In our case, the most suitable for the calculation of the predicted values of ($\widehat{VaR}_{0.01}$ and $\widehat{ES}_{0.01}$ for VI period is the application of the approach with the ordinary differential equations. For this purpose, we consider the following ordinary differential equation:

$$y' = ay \quad (9)$$

Equation (9) can be solved by numerical integration or by deriving an analytical solution, when a is known. It is also possible to calibrate a at different time intervals. An approach for solving equation (9) is given by Marcela Lascsáková. (Marcela Lascsáková (January 2009), Kosice, Slovakia) The analytical solution of equation (9) is: $y(t) = Ce^{at}$

The coefficients a and C are obtained using the application cftool (Curve Fitting tool), which is included in $Matlab$. The equation $y(t) = Ce^{at}$ is used, where t are the analyzed periods, and $y(t)$ will be the values of the parameters on which the Curve Fitting tool in Matlab will be applied. The constants a and C for the estimates of the parameters u, N, $\hat{\alpha}$, $\hat{\sigma}$ are obtained. The obtained results are shown in Table 2.

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Table 2.

	u	N	$\hat{\alpha}$	$\hat{\sigma}$
C	10.68	11.81	1.65	7.336
a	−0.11	0.3653	0.04686	−0.04767

For period **VI** the forecast estimates of the parameters u, N, $\hat{\alpha}$, $\hat{\sigma}$ are calculated as follows:

$u(t=6) = Ce^{at} = 10.68 * e^{(-0.11*6)} = 5.5200$

$N(t=6) = Ce^{at} = 11.81 * e^{(0.3653*6)} \cong 106$

$\hat{\alpha}(t=6) = Ce^{at} = 1.65 * e^{(0.04686*6)} = 2.1857$

$\hat{\sigma}(t=6) = Ce^{at} = 7.336 * e^{(-0.04767*6)} = 5.5112$

Table 3. Peaks over threshold analysis of troy ounce gold values with forecast for period VI

Period	**I**	**II**	**III**	**IV**	**V**	**VI**
u	9.7500	8.2800	7.7600	6.7600	6.3100	5.5200
N	16	26	34	52	73	106
$\hat{\alpha}$	1.7256	1.7831	1.9674	1.9463	2.0909	2.1857
$\hat{\sigma}$	6.7773	6.7549	6.6987	6.0653	5.5656	5.5112
$\hat{\xi}$	0.5795	0.5608	0.5083	0.5138	0.4783	0.4575
$\widehat{VaR}_{0.01}$	50.5243	42.0608	35.8426	35.1084	33.1187	38.2900
$\widehat{ES}_{0.01}$	122.8326	100.5778	78.4929	77.5406	68.3622	76.0869

Subsequently, these parameters are used to find the predicted values of $\widehat{VaR}_{0.01}$ and $\widehat{ES}_{0.01}$ for period **VI**.

The results in Table 3 show that with a probability of 0.01 the positive monthly returns during period VI will exceed 38.29% and that in these cases the average price increase will be 76.09%, i.e. in cases when the monthly return exceeds 38.29%, prices will rise by an average of 76.09% per month.

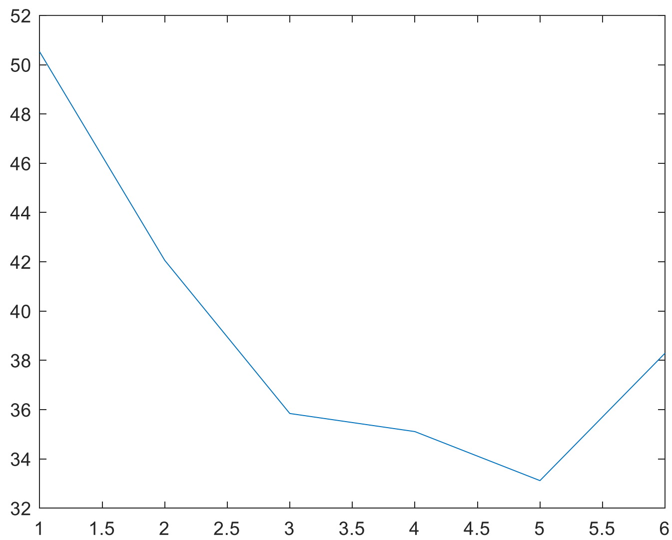

Figure 7. $(\widehat{VaR})_{0.01}$ for periods I – VI

The results of mathematical calculations show a significant risk of increasing the price of gold, but it is worth noting that this risk is long-awaited by financial analysts. Figures 7 and 8 illustrate the time values of $\widehat{VaR}_{0.01}$ and $\widehat{ES}_{0.01}$ at a safety factor of 1% for the analyzed six periods. On the same figures an increase in the values of $\widehat{VaR}_{0.01}$ and $\widehat{ES}_{0.01}$ can be observed during period **VI**. Despite this expectation, in the past there have been periods during which they have been significantly higher.

In recent decades, there has been a positive upward trend of the gold price. The average gold price has risen significantly in recent decades (Table 4) and an increase of 76.09% in period

VI would be much more significant than in the previous periods. Therefore, it should be noted that in the VI period, in the years from 2021 to 2030 there will be a significant risk of a drastic increase in the gold value, which a prerequisite both for economic and subsequent socio-political changes.

Table 4. Mean values of XAU (on monthly data)

Period	**I**	**II**	**III**	**IV**	**V**
USD per Troy Ounce	191.77	393.48	340.20	622.90	1 400.90

Further research is needed to answer the question of whether there is a risk of successive repetitions of such a jump several times in a period of consecutive months. Given that such an event could be the reason for the dynamic changes in the global financial and economic system.

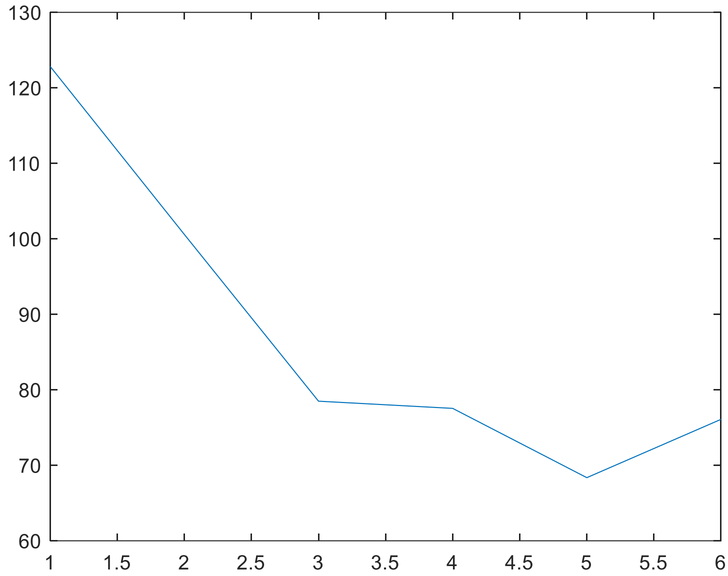

Figure 8. Plot of $\widehat{ES}_{0.01}$ for periods I – VI

CONCLUSION

In recent years, gold has begun to regain its historical significance, which may be followed by an increase in its market price. If this happens for a short period of time and is accompanied with large value increase, then it can be classified as a market risk. This publication includes a brief introduction to the importance of gold for the global financial system and short description of peaks over threshold method, as well as its application in the analysis of gold price market risk. A forecast assessment of the risk value and the expected shortfalls for the period from 2021 to 2030 has been made. Ordinary differential equations and cftool in Matlab were also applied in the calculations of the predicted estimates of VaR and ES.

The results show that:

- In the period between 2021 and 2030 there is a significant risk of a considerable increase in the value of gold;
- Contrary to the previous periods, the market risk of gold will have an increasing trend and gold prices could rise with values which are not observed in recent decades.
- The significant increase of gold value is a prerequisite for both financial and subsequent socio-political changes.

Subject for further analysis is: Study of the risk of several consecutive repetitions of similar jumps in future periods. For this purpose, extremal value theory, analysis of stochastic processes or a combination of both of them could be applied.

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