FRI-ONLINE-1-EEEA-01

STUDYING OF A SERVO SYSTEM IN STATE SPACE

Assoc. Prof. Donka Ivanova, PhD
Department of Automatics and Mechatronics,
University of Ruse "Angel Kanchev"
Tel.: +35982 888 266
E-mail: divanova@uni-ruse.bg

Princ. Assist. Martin Dejanov, PhD
Department of Automatics and Mechatronics,
University of Ruse "Angel Kanchev"
Tel: +359 82 888 747
E-mail: mdejanov@uni-ruse.bg

Abstract: *This paper discusses a way to control a servo system in a state space. For the purpose of the study, a laboratory modular servo system of INTECO Company was used in combination with MATLAB / Simulink for research of different types of control systems. Within the study, a state controller and an observer were synthesized. The operation of the two types of control systems is compared, and a comparative analysis is made with a standard PD regulator. The results of the study show that both regulators meet the quality requirements of the system, and the transient processes are identical, but when using a state observer, the system can be controlled without the need to measure state variables (motor speed).*

Keywords: *State space control, State Space Observer, Laboratory Servo System, MATLAB/Simulink.*

INTRODUCTION

The need to create complex, high-quality control systems and the rapid development of computer technology motivate the application of a new approach to analysis and synthesis of control systems, which is based on the use of mathematical models in the state space. Compared to the classical approach, using input-output models as transfer functions and frequency characteristics, the new approach is revealed:

• State space models are much more suitable for analysis and synthesis of control systems using computer systems, because of an effective methods, algorithms and programs that have been developed;

• The state space model allows uniform analysis and synthesis with identical methods for one-dimensional or complex multidimensional control systems (Tudoroiu, R., 2012) (Tudoroiu, R., 2016).

The purpose of this paper is to investigate a laboratory modular servo system (INTECO, 2021) in the state space by synthesizing a state regulator and state observer.

EXPOSITION

Laboratory Modular Servo System

For the purpose of the research a laboratory modular servo system of the company INTECO presented in fig.1 was used (Modular Servo System, 2013). The servo system consists of several modules mounted on a metal rail and coupled to each other. The modules are a DC motor with independent excitation and tachogenerator, inertia unit, slack, encoder, magnetic brake, reducer and transmission with disk. The laboratory stand allows you to easily and quickly explore different types of control laws. This is achieved by the input-output module "RT-DAC/USB2 I/O board" (INTECO Manual, 2014), which allows real-time exchange of information between the stand and the computer. The company INTECO provides additional toolbox for the MATLAB/Simulink environment, through which various control systems can be developed, including the laboratory system as a virtual unit. To make a connection between the simulation model and the laboratory system, two main software components are used in the Matlab environment: the Real Time Workshop toolkit and the Real Time Windows Target. Thanks to the last two modules, a control program in the C ++ language is generated, which allows real-time control of the system, as well as visualization of the processes directly in the Matlab/Simulink environment (Grepl R., 2011) (Bechar M. at all, 2018).

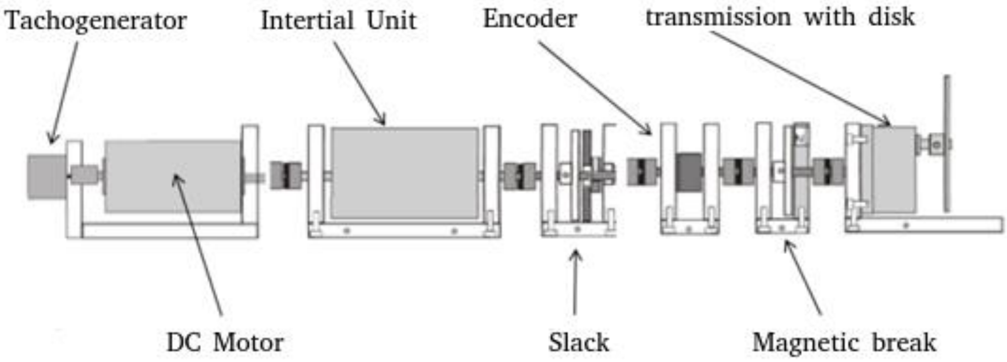


Fig.1. Laboratory modular servo system

Synthesized state space controller

The laboratory system is presented as an object of control, including the following modules: DC electric motor with independent excitation E, reducer P, rotating disk M and power electronic unit (PEU). The controlled values of the object are the angle of rotation y of the motor. An important note must be mentioned that when using the simulation block in Matlab environment, the control effect applied to the object is normalized $|u\ (t)| \leq 1$ (u is a dimensionless quantity). This is due to the fact that usually the output effect of the regulator is given in % or is a dimensionless value in the range from 0 to ± 1. The description of the control system will be based on the dynamic characteristics of the object. If the electromagnetic time constant of the motor and the dynamics of the power electronic unit are neglected, the transfer function of the control system should be in the following form:

$$W_o(p) = \frac{k_M}{p(T_M p+1)}, \tag{1}$$

where $k_M = 190\ rad/s$ and $T_M = 1\ s$ are the coefficient of the motor and the electromechanical time constant.

It is assumed that in the process of operation the setpoint r gets non-zero constant values. It is required that the overshoot should not exceed 10% and the settling time (at 2 percent zone) must be less than 1.5 s.

For the purpose of the research, a state regulator has been developed, and the pole placement method for synthesis is used to determine its parameters (Unbenhauen H., 2009) (Ackerman, 2021). The equation (1) presented in the form of a transfer function will be transformed into a state space using a phase-coordinate canonical form. The equation describing the object is presented below:

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = C^T x(t), \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T_M} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{k_M}{T_M} \end{bmatrix}, \quad C^T = [1 \quad 0],\ x_1(t) = y(t),\ x_2(t) = \frac{dy(t)}{dt}$$

The angle of rotation and the speed are selected as state variables. Marked as x_1 the angle of rotation of the motor, which is measured with a photoelectric raster converter (encoder) and x_2 the angular velocity of the motor, which is measured by a tachogenerator. An important prerequisite for synthesizing a state regulator is to be able to measure state variables. Taking the transfer

 Copyrights© 2021 ISBN 978-954-712-864-4 (Print)

function (1) it can be concluded that the object is completely controllable. The control law has the following form:

$$u(t) = k_1\big(r(t) - x_1(t)\big) - k_2 x_2(t). \tag{3}$$

The values of the controller coefficients k_1 and k_2 are determined by the pole placement method. In accordance with the transient process quality characteristics, a characteristic equation is chosen in the standard Butterworth form (Lehov G., & Ivanova D., 2019):

$$H^*(p) = p^2 + 1.4\omega_o p + \omega_o^2 = 0 \tag{4}$$

For the parameter ω_o we choose $\omega_o = 4.5\ s^{-1}$. The matrix of the closed loop system is determined by the equation:

$$A - Bk^T = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T_M} \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{k_M}{T_M} \end{bmatrix} [k_1 \quad k_2] = \begin{bmatrix} 0 & 1 \\ -\frac{K_M}{T_M}k_1 & -\left(\frac{1}{T_M} + \frac{k_M}{T_M}k_2\right) \end{bmatrix} \tag{5}$$

The resulting matrix is in accompanying form and the characteristic equation of the closed loop system can be written as:

$$H(p) = p^2 + \left(\frac{1}{T_M} + \frac{k_M}{T_M}k_2\right)p + \frac{K_M}{T_M}k_1 = 0. \tag{6}$$

After equating the coefficients in front of the same degrees of the characteristic equations (4) and (6), the coefficients in the control law are obtained as follow:

$$k_1 = \frac{\omega_o^2}{k_M}T_M = 0.11\ rad^{-1}\ ,\ k_2 = \frac{1.4\omega_o T_M - 1}{k_M} = 0.028\ (rad/s)^{-1}.$$

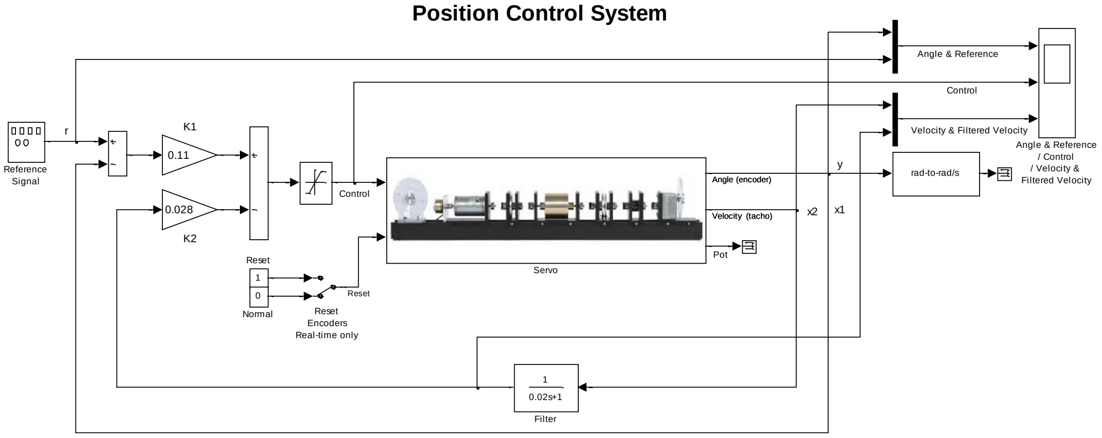
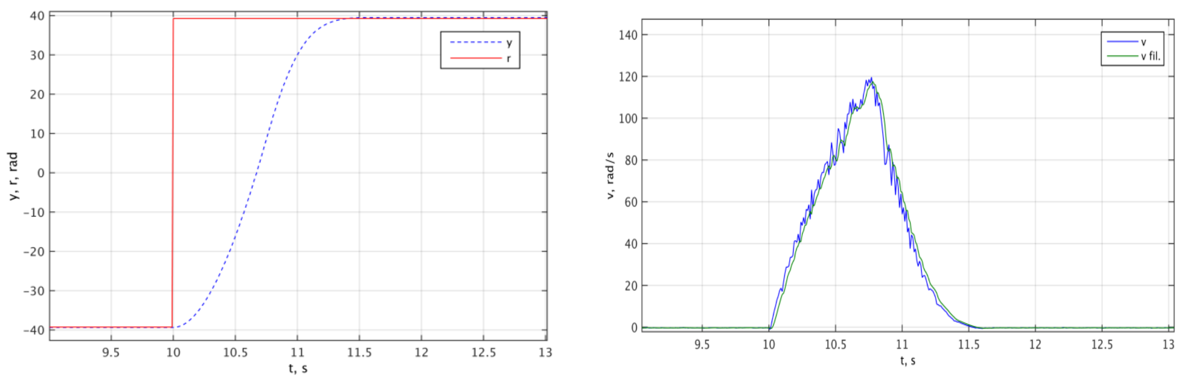


Fig.2. Model of the electromechanical state control system with a state controller



a) b)

Fig.3. Process variable 'y' (a) and angular velocity (b) for two variants filtered and unfiltered

When measuring the signal from the tachogenerator, it is very noisy and therefore when implementing the control it is advisable to filter this signal. A first order lag function with time constant T_f = 0.02s is included as a filter. Fig.2 shows the simulation model of the electromechanical control system together with the coefficients of the state controller, shown as two "Gain" modules and the filter unit.

Fig.3 shows the process variable y (angle) and the angular velocity (velocity), obtained from the experiment with the laboratory control system with a given setpoint of r = 25π/2. For the overshoot and the settling time of the system the following parameters are obtained: σ = 0.02% and t_p = 1.23s. The synthesized system satisfies the quality requirements.

Synthesize state space observer

To implement the control law (3) it is necessary to have information about the angular position and angular velocity of the motor. Speed measurements with the tachogenerator can be avoided by using a state-space observer. The equation of a full-order state-space observer has the following form:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + l\big(y(t) - c^T\hat{x}(t)\big) \tag{7}$$

The input values of the observer are the output value y(t) of the object and the control output u(t). The output of the observer is the vector $\hat{x}(t) = [\hat{x}_1(t) \quad \hat{x}_2(t)]^T$, which represents the estimate of the state vector x(t). The variable $l(t) = [l_1(t) \quad l_2(t)]^T$ denotes a vector whose values are determined in the synthesis of the observer. The state observer can be written as following: $\dot{\hat{x}}(t) = (A - lc^T)\hat{x}(t) + Bu(t) + ly(t)$.

The coefficients l_1 and l_2 of the observer can be determined by the pole placement method due to the fact that the synthesis of the state observer is mathematically equivalent to the synthesis of the state controller (duality). In the synthesis of the observer we will also use the standard form of Butterworth with characteristic equation (4), as to ensure approximately twice the speed of the observer compared to the closed loop system, the value of the parameter ω_o will be twice as large as determined in previous section and the value should be $\omega_o = 9s^{-1}$.

The observer matrix is the following:

$$A - lc^T = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T_M} \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} [1 \quad 0] = \begin{bmatrix} -l_1 & 1 \\ -l_2 & -\frac{1}{T_M} \end{bmatrix}$$

and the characteristic equation of the observer is determined as follow:

$$H_H(p) = det\big(pI - (A - lc^T)\big) = p^2 + \left(\frac{1}{T_M} + l_1\right)p + \left(\frac{1}{T_M}l_1 + l_2\right) = 0 \tag{8}$$

In the characteristic equations (4) and (8) coefficients are equalized, whereby

$$l_1 = 1.4\omega_o - \frac{1}{T_M} = 11.6\, s^{-1},\ l_2 = \omega_o^2 - \frac{1}{T_M}l_1 = 69.4\, s^{-2}. \tag{9}$$

In this case, only the angle of rotation y(t) of the motor needs to be measured with a sensor. The equation of the state observer and the control law defined in previous section must be realized in the control device, where instead of the state variables $x_1(t)$ and $x_2(t)$ their estimates are used $\hat{x}_1(t)$ и $\hat{x}_2(t)$:

$$\begin{bmatrix} \dot{\hat{x}}_1(t) \\ \dot{\hat{x}}_2(t) \end{bmatrix} = \begin{bmatrix} -l_1 & 1 \\ -l_2 & -\frac{1}{T_M} \end{bmatrix} \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_M}{T_M} \end{bmatrix} u(t) + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} y(t), \tag{10}$$

$$u(t) = k_1\big(r(t) - \hat{x}_1(t)\big) - k_2\hat{x}_2(t).$$

Fig.4 shows a simulation model of a control system including a model of the electromechanical system – block "Servo", as well as the state observer included in the system, presented with the block "Observer". The presented simulation model connects with the real servo system and drives it. The information about the current state of the variables is visualized in real time on the two blocks of type "Scope". The observer tuning parameters are calculated on the basis of equation (8) and represented by equation (9). In order for the observer to be implemented within

 Copyrights© 2021 ISBN 978-954-712-864-4 (Print)

the simulation model, it is necessary to present it in state space, such as equation (11) and the matrices (12) that are presented below:

$$\dot{\hat{x}}(t) = A_o\hat{x}(t) + B_o v(t) \qquad (11)$$

$$y_o(t) = C_o\hat{x}(t) + D_o v(t)$$

where: $v = \begin{bmatrix} u \\ y \end{bmatrix}$, $y_o = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$, $A_o = \begin{bmatrix} -11.6 & 1 \\ -69.4 & -1 \end{bmatrix}$, $B_o = \begin{bmatrix} 0 & 11.6 \\ 190 & 69.4 \end{bmatrix}$,

$$C_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ D_o = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (12)$$

The experiment with the laboratory control system at setpoint r = 25π/2 was conducted. During the research it turned out that the processes obtained with a state controller and a system with an observer were identical. This confirms the thesis that the observer manages to fully compensate the need to measure one of the state variables. Figures 5a and 5b show the characteristics of these processes.

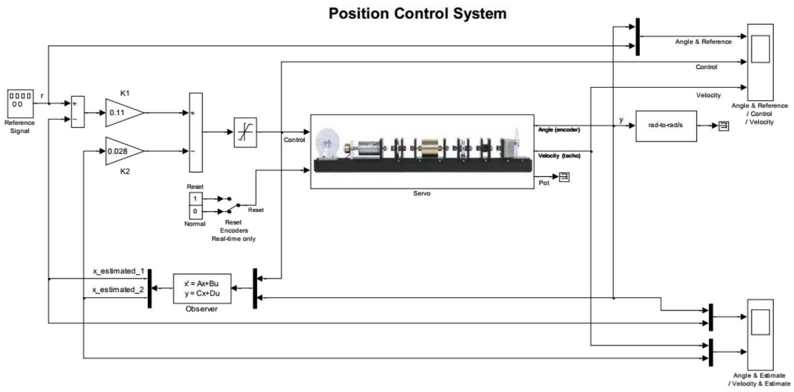
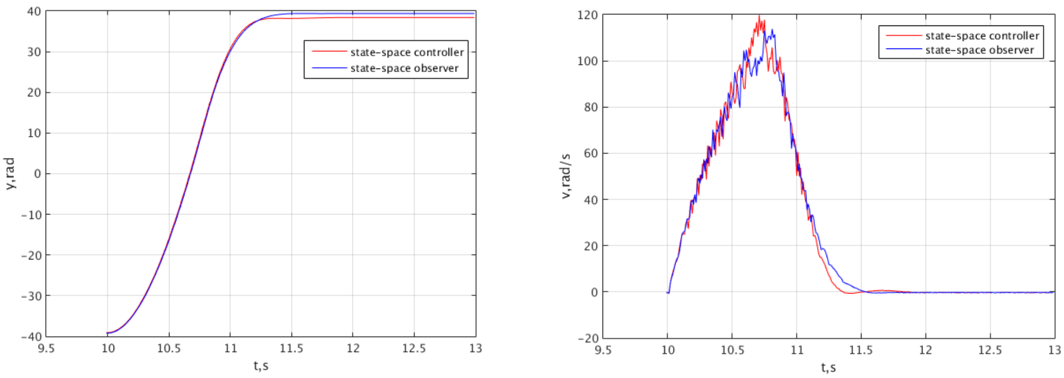


Fig.4. Model of the electromechanical control system with a state observer

It is obvious that the estimates formed by the observer are very close to the respective real values, and high quality of the transient processes is ensured. In this research, a variant of the system by means of a PD regulator was also considered. The controller is set with optimal parameters.



a) b)

Fig.5. (a) transient process for the angle of rotation with and without observer (b) transient characteristics for speed with and without observer

A comparison was also made between the operations of the three types of control structures. The transient characteristics obtained in the study are shown in Fig.6. It can be seen that the operation of a controller in the state space with or without an observer handles the process faster than the PD controller. As well the settling time of both controllers is better performed compared to the optimally tuned PD controller.

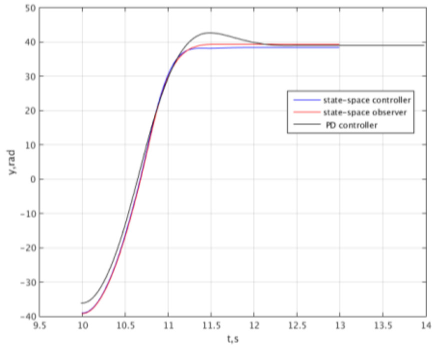


Fig.6 Transition process for the three types of control structures

CONCLUSION

In the process of research, a system for controlling the angle of rotation of a DC motor with independent excitation was synthesized. As well a state observer was synthesized. The main advantage of using an observer is the ability to control the angle of rotation on the basis of the speed estimation. This allows control of the system without the need for speed measurement with a tachogenerator and avoids its noisy measurements. Both types of control systems based on a state controller are studied. In the process of research it turned out that the estimates formed by the observer are very close to the measurementsobtained with a state controller. In both cases, the synthesized system satisfies the predefined quality requirements. From the experiments turns out that the state controllers have better quality indicators than the standard control laws, such as PD controller.

ACKNOWLEDGMENTS

The research is supported by a contract with Ruse University "Angel Kanchev" under NSF–2021 - EEA – 05.

REFERENCES

Bechar M., Hazzab A., Habbab M., Lakhdari L., Slimi M. (2018). Real-Time Control of AC Machine Drives Using RT-LAB Package. In: Hajji B., Tina G.M., Ghoumid K., Rabhi A., Mellit A. (eds) Proceedings of the 1st International Conference on Electronic Engineering and Renewable Energy. ICEERE 2018. Lecture Notes in Electrical Engineering, vol 519. Springer, Singapore.

Grepl R., (2011). "Real-Time Control Prototyping in MATLAB/Simulink: Review of tools for research and education in mechatronics," 2011 IEEE International Conference on Mechatronics, Istanbul, 2011, pp. 881-886, doi: (9) 10.1109/ICMECH.2011.5971238.

Lehov G., Ivanova D. (2019) Control Theory - Part 1, Academic Publishing House of the University of Ruse, Ruse, 2019 (Оригинално заглавие: 1. Лехов, Г., Д. Иванова. "Теория на управлението" - Част 1, Академично издателство на Русенския университет, Русе, 2019)

Modular Servo System (2013). User's Manual, 2013.

Tudoroiu R., W. Kecs, M. Dobritoiu, N. Ilias, S. Casavela, N. Tudoroiu, (2016) Real-Time Implementation of DC Servomotor Actuator with Unknown Uncertainty using a Sliding Mode Observer, Proceedings of the Federated Conference on Computer Science and Information Systems 2016 Vol. 8, pp. 841–848, ISSN 2300-5963.

Tudoroiu R., (2012) Conceiving and Implementing Applications using Real-Time UML, PhD Thesis, Cluj-Napoca Technical University, Romania.

Unbenhauen H., (2009) Control Systems, Robotics and Automation vol. 8, Advanced control systems – II, 2009, Encyclopedia of Life Support Systems.

Ackerman, (2021) https://en.wikipedia.org/wiki/Ackermann%27s_formula

INTECO, (2021) www.inteco.com.pl (Accessed on 06.10.2021)

INTECO Manual, 2014 http://www.inteco.com.pl/wp-content/uploads/2014/05/RTDAC_USB2.pdf

 Copyrights© 2021 ISBN 978-954-712-864-4 (Print)