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MODAL ANALYSIS AND FREE VIBRATION TEST PERFORMING AND COMPARING FOR A SIMPLY SUPPORTED BEAM WITH A CRACK

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Abstract: *Modal analysis is a linear procedure that calculates natural frequencies through stiffness and mass matrices. But, if there are defects in structures, processes become non-linear. For an example, if a beam has a crack, the stiffness is not constant across the section. This is expected to lead to non-constant moment of inertia over the time when bending vibrations occurs. It is reasonable that an impact may arise to turn the problem into strongly nonlinear. In this paper, the natural frequencies and corresponding modal shapes is firstly obtained for a non-cracked simply supported beam through analytical relations. After that, this is realized for a non-cracked and for a cracked FEM model* ***by two ways****: a modal analysis based on the stiffness and mass matrices – neglecting the stiffness nonlinearity and the contact interactions;* ***and a numerical free vibration test – accounting the stiffness nonlinearity and the contact interactions****. The numerical free vibration test uses the Fourier transform to extract the vibration harmonics. Finally, the results are compared and conclusion are claimed.*

Keywords: *Modal Analysis, Natural Frequencies, Natural Shapes, Beam, Crack, Finite Element Method, Stiffness Nonlinearity, Contact Interactions, Impact, Impact Test, Fourier Transform, Harmonic Analysis, Timediagrams, Spectrograms, Vibration Acceleration, Vibration Velocity, Vibration Displacement.*

1. INTRODUCTION

The **modal analysis** of beams provides insights into their **natural frequencies** and **mode shapes**, which are essential for understanding their dynamic response and assessing their health. In (Stoyanov, S., 2017) is presented an investigation of the natural and resonant frequencies of a **cantilever beam**. In (Stoyanov, S., 2018)., theoretical and experimental researches of the resonant frequencies of a **plane frame** is performed. The results from the experimental setup created are compared to the theoretically obtained results and it is found that the difference is under 6%.

Most of the members of engineering structures operate under loading conditions, which may cause damages or cracks in overstressed zones (Quila, M., Sarkar, S., 2024). Therefore, the study of cracked beams is a significant area of research in structural engineering and material science. Cracks can severely affect the structural integrity and dynamic behavior of beams, which are critical components in various engineering applications, including bridges, buildings, and machinery. The presence of cracks causes changes in the physical properties and dynamic response characteristics of beams (Jagdale, P., Chakrabarti, M., 2013).

The presence of cracks in beams can lead to catastrophic failures if not detected and addressed in time. Traditional methods for inspecting and maintaining structural integrity often fall short in identifying internal defects such as cracks. Therefore, there is a need for advanced techniques that can accurately predict the effects of cracks on the modal parameters of beams. Theoretical modal analysis not only aids in understanding the fundamental dynamics of cracked beams, but also contributes to the development of effective monitoring and diagnostic techniques. Damage identification plays a vital role by providing timely damage assessment, which improves safety and maintains high performance and reliability for civil structures (Subhasmita, P., 2019).

The aim of this investigation is to find out the differences in the natural frequencies and mode shapes for several beam models, including a homogamous beam and a beam with a crack. To achieve this aim, the following tasks are defined:

Task 1: Obtaining the first five natural frequencies and mode shapes of an **ideal** (without any defects) pined-rolled simply supported beam through analytical relations based on the theory of the Euler and Timoshenko beams.

Task 2: Obtaining the first five natural frequencies and mode shapes of a **FEM model** of the same ideal beam from **Task 1** in two cases:

– the supports are applied on the beam mid line, as it is supposed in the Euler and Timoshenko beam theory;

– and the supports are applied on the entire beam's surface, as it can be in the real fiscal couplings.

Task 3: Modeling of **a crack** into the above FEM model and performing linear modal calculations by stiffness and mass matrix. After that, performing a numerical impact test, obtaining and analyzing the free vibrations to determine the vibration spectrogram.

The object of this investigation is a straight beam with length of 200 mm. The beam has a solid quadratic cross section with dimensions 10x10mm. At the left end, the beam is pinned (hinged). At the right end, the beam is rolled. The "ideal" beam is homogenous, and has no holes or defects, like cracks for example. The "cracked" beam has a crack with depth of 2.5 mm and width of 0.002 mm, and ends with a round with radius of 0.001mm. The crack location is at the middle of the beam, but the impact force applying point varies with consideration of the nodes in the natural mode shapes of the beam. The crack is on the bottom beam side, and the impact force application point is on the top beam side.

2. FREQUENCY EQUATIONS OF PINED-ROLLED SIMPLY SUPPORTED BEAM

To derive the equations for **natural frequencies** of beams under the **Euler-Bernoulli** and **Timoshenko** beam theories, let's review the governing equations and assumptions in each theory.

In Euler-Bernoulli beam theory, cross-sections of the beam are assumed to remain perpendicular to its neutral axis and do not undergo any deformation during bending. This assumption neglects shear deformation and rotary inertia, making this theory **more accurate for slender, long beams.**

The equation of motion for transverse vibrations in a **Euler-Bernoulli beam** is given by (Quila, M., Sarkar, S., 2024):

$$\frac{\partial^2}{\partial t^2}\big(\rho A w(x,t)\big) + \frac{\partial^4}{\partial x^4}\big(EI w(x,t)\big) = 0, \tag{1}$$

where: $w(x,t)$ is the transverse displacement, E is the Young's modulus, I is the area moment of inertia, ρ is the material density, and A is the cross-sectional area.

The **Timoshenko beam** theory accounts for:

– Shear deformation, addressing the relative displacements in cross-sections,

– Rotary inertia, which considers the rotational motion of cross-sections.

This leads to the following modified equation of motion (Zhou, B., Bingham, H., Shao, Y., 2024):

$$\frac{\partial^2}{\partial t^2}\big(\rho A w(x,t)\big) + \kappa G A \frac{\partial^2 w(x,t)}{\partial x^2} = EI \frac{\partial^4 w(x,t)}{\partial x^4}, \tag{2}$$

where: G is the shear modulus, κ is the shear coefficient.

The main difference in frequencies between the two models arises from the inclusion of shear deformation and rotary inertia in Timoshenko theory. These factors have a pronounced effect on shorter, thicker beams and produce slightly lower natural frequencies compared to the Euler-Bernoulli model, especially in higher modes.

According to Eq. 1 and Eq. 2, the first five natural frequencies and mode shapes of the ideal beam investigated are calculated and presented on Fig. 1 and Table 1.

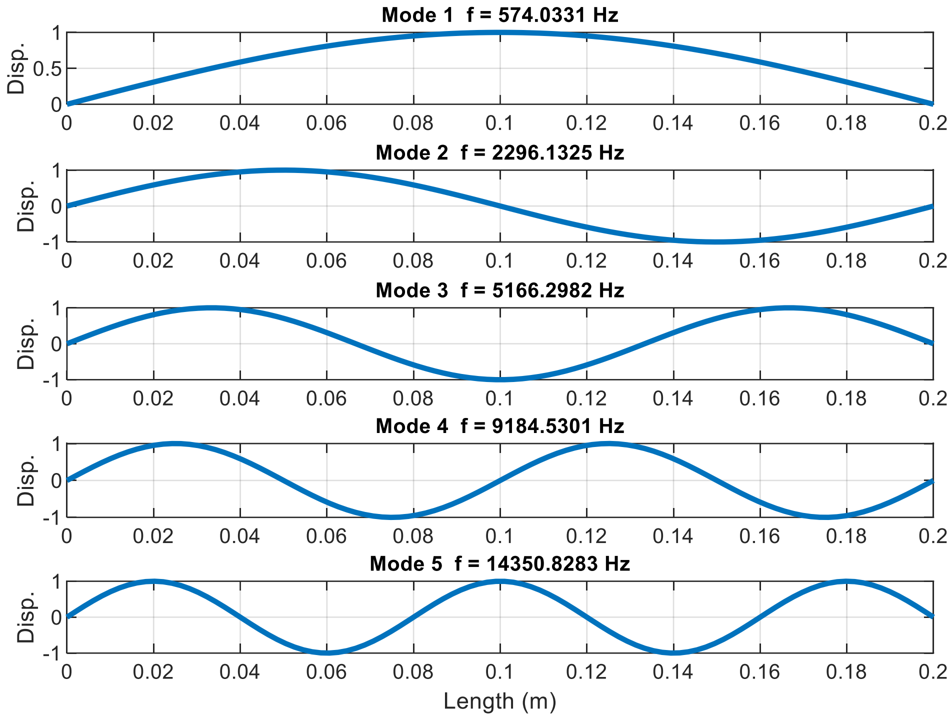

Fig.1. Natural frequencies and mode shapes according to Euler-Bernoulli theory

Table 1. Analytically obtained results for the ideal beam

Mode №	Natural frequency, Hz			
	Euler-Bernoulli Hz	Timoshenko Hz	Absolute difference Hz	Relative difference %
1	574	572	2	0.35
2	2296	2259	37	1.64
3	5166	4985	181	3.63
4	9185	8638	547	6.33
5	14351	13090	1261	9.63

3. FINITE ELEMENT LINEAR MODAL ANALYSIS OF THE IDEAL BEAM

Finite element analysis gives the opportunity to obtain results not only for a beam that is supported on its mid line, but also for supporting devices, which are mounted on the whole side section area. So, a comparison of the results obtained for these two cases is presented on the Fig. 2 and Fig. 3.

a **b**

Fig. 2. The first natural frequency and mode shape of the investigated beam:
a – for mid line supports (572Hz), b – for whole side supports (888 Hz)

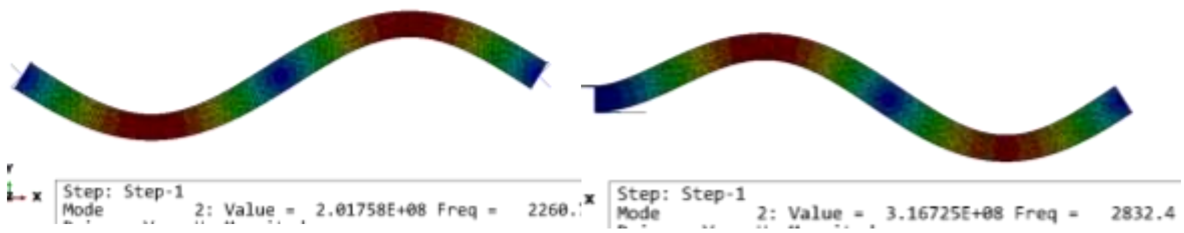

Fig. 3. The second natural frequency and mode shape of the investigated beam:
a – for mid line supports (2260 Hz), b – for whole side supports (2832 Hz)

It must be taken a decision which supporting scheme to be used for the basis results that will be compared with the results from the cracked beam. The choice is made on **the middle line supported case**, as this is the analytically comparative case, and aliso because this is the more commonly used option in general.

4. FINITE ELEMENT LINEAR MODAL ANALYSIS OF THE CREAKED BEAM

On Fig. 4 are presented the first five natural frequencies and mode shaped of the cracked beam, obtained through the modal analysis performed. One can notice that the crack may be opened or closed in the different mode shapes. In other words, for a given mode shape, the crack may be located at a node, and also it can be located in an antinode. The values of the natural frequencies are listed in Table 2, and the location of crack is also noted to compare the different cases. **The four natural frequency and mode shape corresponds to axial deformations and are not obtained in the analytical results presented above (Table 1).**

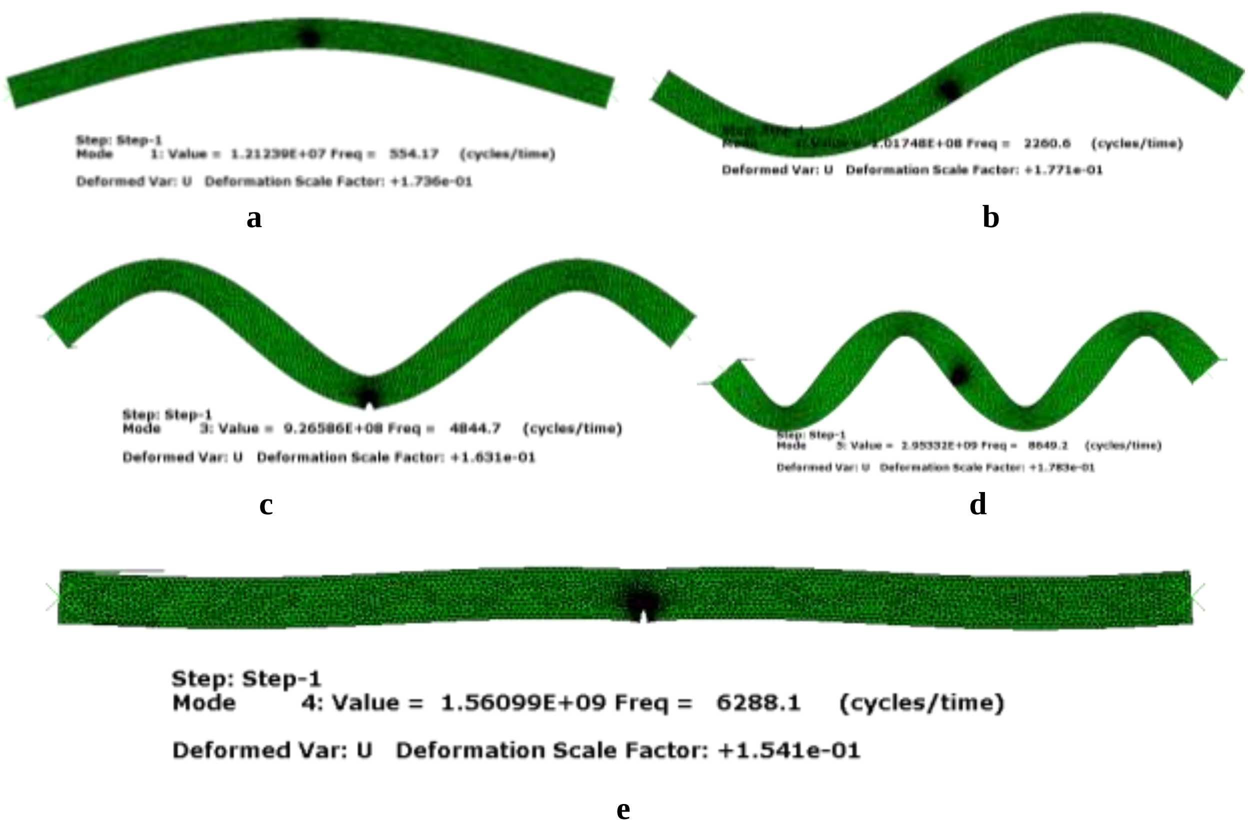

Fig. 4. The first five natural frequencies and mode shaped of the cracked beam:
a – the first natural frequency (554 Hz) and mode shape, b – the second natural frequency (2261 Hz) and mode shape, c – the third natural frequency (4845 Hz) and mode shape, d – the five natural frequency (8649 Hz) and mode shape, and e – the four frequency (6288 Hz) and shape

5. IMPACT TEST MODAL ANALYSIS

The impact force is applied at 140 mm from the left beam end, attempting to avoid the natural nodes (Fig. 1). The spectrogram obtained though Fast Fourier Transform for **the ideal beam free vibration acceleration** is shown on Fig. 5. The application impact force point is close to one of the nodes of the third mode shape, so this mode shape is excited very slightly. As a result, on Fig. 5 can be seen four peaks, as follows: **the first natural frequency of 567 Hz, the second –2267 Hz, the four – 8666 Hz, and the five – 13166 Hz.**

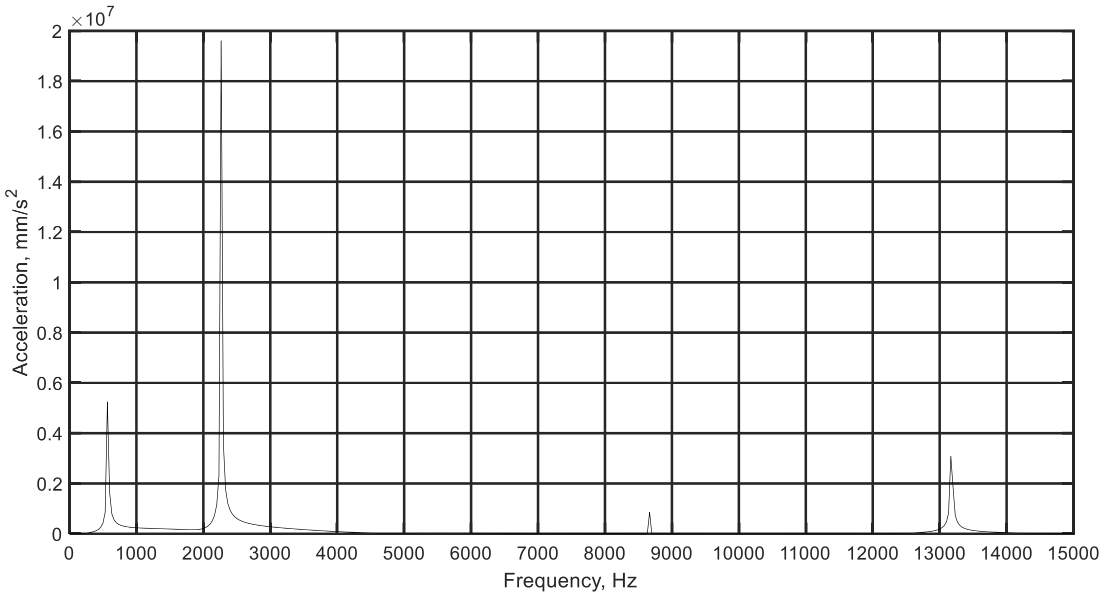

Fig. 5. A spectrogram from the impact modal test on the ideal beam

Other point also can be used as an impact force application point. For example, as the application point may be used the middle beam point at 100 mm. But this is an inappropriate option for this investigation, and the reason can be seen on Fig. 6. This investigation is interested from the nonlinear effects due the crack presence. On Fig. 6 is presented **the spectrogram for the cracked beam**. It can be seen that there are **two subharmonics** (at 12400 Hz and at 13532Hz) around the five natural frequency (at 12966Hz) in difference with the spectrogram of the ideal beam. As is known in scientific fields of nonlinear oscillations, the occurrence of such subharmonics is **an indication of the presence of nonlinearity** (Stoyanov S., 2015).

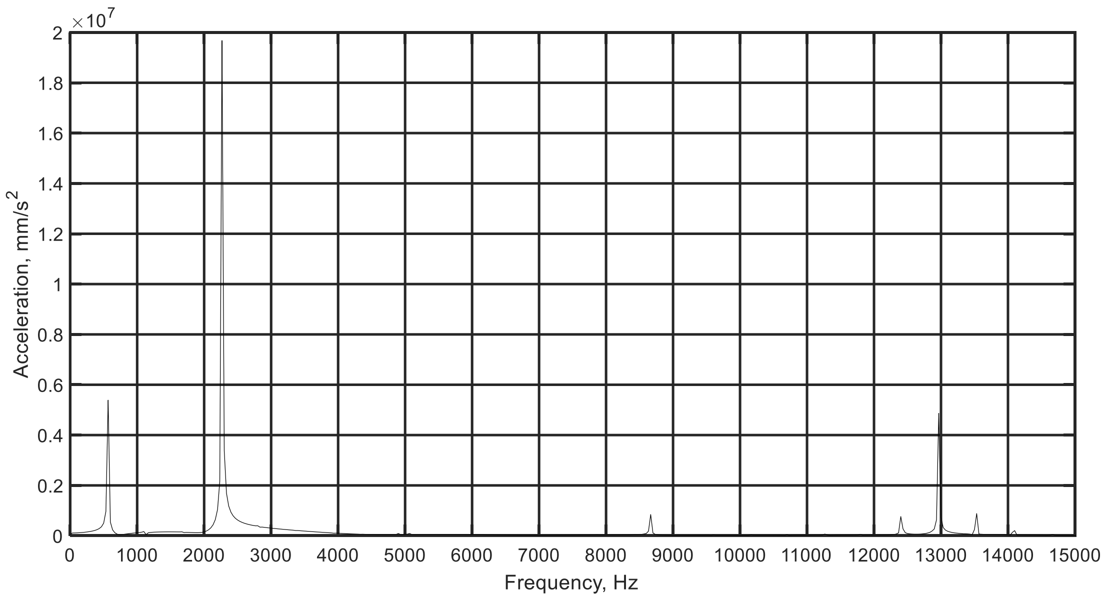

Fig. 6. A spectrogram from the impact modal test on the cracked beam

6. RESULTS AND CONCLUSIONS

The obtained results are summarized in Table 2.

Table 2. Results from the modal and impact tests

Natural mode №	Natural frequency					Crack Position
	Ideal beam, Linear modal Hz	Cracked beam		Ideal vs Cracked, modal		
		Linear modal Hz	Impact test Hz	Absolute difference Hz	Relative difference %	
1	**572**	**554**	**567**	**18**	**3**	**In antinode**
2	2261	2261	**2267**	0	0	In node
3	**4990**	**4845**	**–**	**145**	**3**	**In antinode**
4	**6332**	**6288**	**–**	**44**	**0.7**	**In antinode**
5	8650	8649	8666	1	0.01	In node

The planed tasks of this investigation are performed successfully. With the help of Table 2, one can observe that if the crack is located at mode shape antinode, these is a deference between the natural frequencies of the ideal and the cracked beam investigated. Also, the impact modal test performed indicates the presence of nonlinearity trough two subharmonics in the free vibration acceleration spectrogram of the cracked beam. The presented in Table 2 date, show that there are still questions to clarify and the investigation should be continued and enhanced.

7. REFERENCES

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