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ENGEL'S CURVE: MODELING THE RELATIONSHIP BETWEEN CONSUMPTION AND INCOME

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***Abstract:** In the 19th century, the German statistician Ernst Engel conducted groundbreaking research on consumer behavior, particularly focusing on how income impacts spending across different categories. His findings, published in 1857, established a foundation for future studies in this area. Engel discovered that there exists an inverse relationship between income and the consumption of certain goods, which laid the groundwork for modern consumer behavior theory. Based on that theory, the current paper examines the relationship between consumption of three commodities and money income. A correlation analysis is performed, based on which inverse models are constructed for the consumption of each of the three commodities. The models are tested for adequacy and significance of coefficients as well as normality of residuals. A comparative analysis of the results is done.*

***Keywords:** Consumer Behavior, Engel's Curve, Income-Expenditure Relationship, Inverse Model.*

INTRODUCTION

The Engel curve is a basic concept in microeconomics that studies the relationship between household income and consumption. Named after the 19th century German economist Ernst Engel, it serves as an important tool for understanding consumer behaviour and economic dynamics. It illustrates how changes in household income affect spending on different categories of goods and services. It distinguishes between basic goods, whose consumption remains relatively constant across income levels, and luxury goods, which have a more significant increase in consumption as income rises. One of the key observations derived from the Engel Curve is Engel's Law, which states that as household income increases, the share of income spent on food decreases [1]. This law highlights the importance of food as a basic necessity and provides a measure for analyzing consumption patterns.

The Engel curve provides information on the elasticity of consumer demand and price sensitivity for various goods and services. Businesses can use this analysis to determine optimal pricing strategies, develop promotional offers, and build marketing campaigns that maximize revenue and profit. Changes in expenditure and income patterns provide information on changes in living standards and the evolution of the economy over time and space, revealing vulnerable groups of consumers who spend a significant proportion of their income on basic goods and services. This information serves as a tool for measuring well-being and economic development [2].

The construction of the Engel curve involves the analysis of household expenditure data collected from surveys, diaries or national statistics. Economic methods, such as regression analysis or Engel curve estimation models, are used to estimate the relationship between income levels and consumption patterns across different expenditure categories, provide views on the income elasticity of demand, which measures how changes in income affect demand for different goods. Goods with high income elasticities, such as luxury goods, exhibit steep Engel curves. This is an indicator of a significant increase in consumption as income increases, which guides

the design and implementation of various economic policies. Regression analysis is the usual method for estimating Engel curves. By regressing household expenditures against income, the income elasticity of various categories of expenditures can be determined and Engel curves can be plotted for various goods and services. Although often depicted as a linear relationship between income and expenditure, actual data may show non-linear effects. In such cases, advanced econometric techniques such as quantile regression or polynomial regression are used to account for these complexities and obtain more accurate estimates [3].

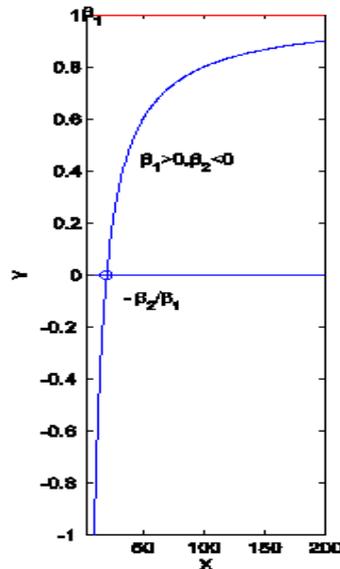


Figure 1. The Engel curve: an inverse model for consumption of a good and income (source: see [4])

Figure 1 shows the relationship between consumption of a good (Y) and income (X). The figure shows that some goods have the following properties:

- 1) There is an income threshold below which this good is not purchased and this level is equal to $-\beta_2/\beta_1$.
- 2) There is an upper level of consumption of this good that is not exceeded - regardless of income. This level coincides with the asymptotic value β_1 .

The inverse model of Figure 1 is appropriate for describing the consumption of such goods [4].

The inverse model has the form

$$Y_i = \beta_1 + \beta_2 \left(\frac{1}{X_i} \right) + u_i. \quad (1)$$

This model has the following property: when X tends to infinity, $\beta_2(1/X)$ will tend to zero and Y will tend to its asymptotic value β_1 . Hence, using the inverse models, we can obtain the asymptotic value of the dependent variable Y with an unbounded increase in the independent variable X .

This paper examines the relationship between consumption of three commodities and money income, on average, per person in the household. For this purpose, data from the National Statistical Institute (NSI) on a quarterly basis for the period 2016-2023 [5] are used. Using SPSS, correlation analysis is performed on the sample data, based on which inverse models are constructed. The models are checked for adequacy and significance of coefficients as well as for normality of residuals. A comparative analysis of the results is performed.

EXPOSITION

A correlation matrix of the variables studied was constructed (Table 1). It shows that the Pearson correlation coefficients for the relationship between money income on average per household person X and average consumption per household person of commodities A (bakery

products, kg), B (eggs, pcs) and C (potatoes, kg) are -0.937, 0.654, 0.199 respectively. The correlation coefficients between money income and consumption of commodities A and B , respectively, are significant (with probabilistic guarantees close to 100%) and indicate the presence of a strong correlation, while we treat the correlation of money income with consumption of commodity C as low and insignificant. From the correlation coefficient between money income and consumption of good A , which is negative, we can infer that as income increases, consumption of bakery products will decrease. The correlation coefficient between money income and consumption of commodity B is positive and we assume that as income increases, consumption of eggs will also increase.

Table 1. Correlation Matrix

		X – Income	A - Consumption	B - Consumption	C - Consumption
X - Income	Pearson Correlation	1	-0.937	0.654	0.199
	Sig. (2-tailed)		0.00	0.00	0.275
A - Consumption	Pearson Correlation	-0.937	1	-0.754	-0.183
	Sig. (2-tailed)	0.00		0.00	0.315
B - Consumption	Pearson Correlation	0.654	-0.754	1	0.006
	Sig. (2-tailed)	0.00	0.00		0.974
C - Consumption	Pearson Correlation	0.199	-0.183	0.006	1
	Sig. (2-tailed)	0.275	0.315	0.974	

To describe the relationship between the consumption of each of the products A and B and money income, inverse models of the form (1) are constructed. Table 2 shows summary statistics for the two models. The table shows that the adjusted coefficients of determination for the models (*Adjusted R Square*) are 0.907 and 0.496 for products A and B , respectively.

Table 2. Models Summary Statistics

	R	R Square	Adjusted R Square	Std. Error of the Estimates
A – Consumption Model	0.952	0.907	0.907	0.420
B – Consumption Model	0.716	0.513	0.496	1.793

ANOVA tables are made for both models (see Table 3), which can be used to check the adequacy of the regression models. Both models are adequate to the existing significant relationships between the variables with probabilistic guarantees close to 100%.

Table 3. ANOVA tables

		Sum of Squares	df	Mean Square	F	Sig.
A – Consumption Model	Regression	51.809	1	51.809	293.424	0.00
	Residual	5.300	30	0.177		
	Total	57.110	31			
B – Consumption Model	Regression	101.411	1	101.411	31.538	0.00
	Residual	96.464	30	3.215		
	Total	197.875	31			

Table 4 provides information on the coefficients of the models and their significance.

Table 4. Model Coefficients

		Unstandardized B	Coefficients Std. Error	Standardized Coefficient Beta	t	Sig.
A – Consumption Model	1 / X-Income	9372.673	547.331	0.952	17.124	0.000
	(Constant)	14.729	0.334		44.164	0.000
B – Consumption Model	1 / X-Income	-13112.993	2334.971	-0.716	-5.616	0.000
	(Constant)	45.852	1.423		32.227	0.000

For product A:

$$\hat{Y}_i^A = \hat{\beta}_1^A + \hat{\beta}_2^A(1/X_i) = 14.729 + 9372.673(1/X_i)$$

From this model it can be concluded that as income rises, consumption of product A falls. The constructed inverse model shows that the minimum level of bakery products consumption is 14.729 kg per person of the household.

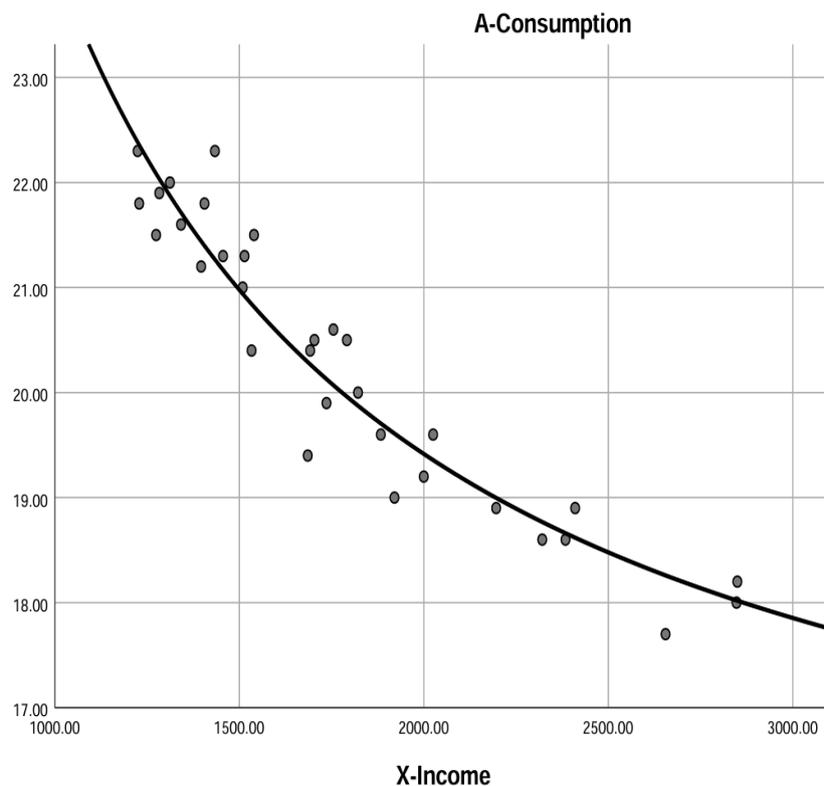


Figure 2. Inverse model for the consumption of product A

For product B:

$$\hat{Y}_i^B = \hat{\beta}_1^B + \hat{\beta}_2^B(1/X_i) = 45.852 - 13112.993(1/X_i)$$

In the case of product B, we can expect that consumption will increase as income increases. The constructed inverse model accordingly shows that the income threshold below which eggs are not purchased is $-\beta_2/\beta_1 = 285.985$ BGN per person in the household.

On the basis of the conducted models, the inverse model was found to describe well the consumption data for bakery products (Figure 2) and eggs (Figure 3).

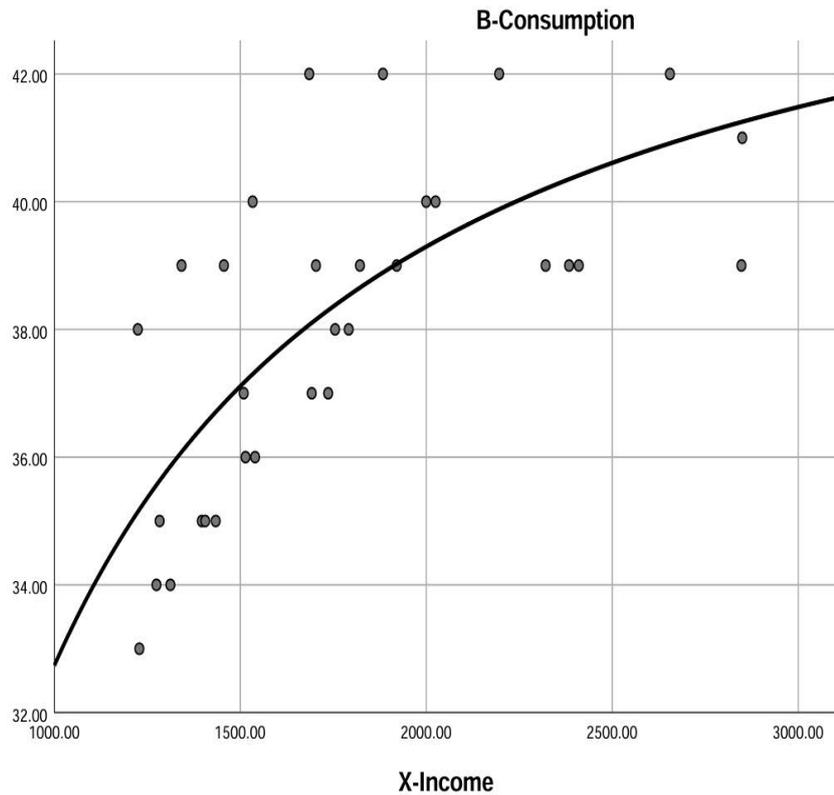


Figure 3. Inverse model for the consumption of product *B*

A test for normality of residuals is performed (see Table 5), which shows that with a 98% probabilistic guarantee for product A, the null hypothesis of normality of residuals is rejected, while for product B the empirical distribution of residuals is close to the normal theoretical distribution.

Table 5. Kolmogorov-Smirnov Test for normality of residuals

		Error for good A	Error for good B
N		32	32
Normal Parameters	Mean	0.00000	0.00000
	Std. Deviation	1.76401328	0.41349529
Most Extreme Differences	Absolute	0.171	0.092
	Positive	0.171	0,68
	Negative	-0.104	-0.092
Test Statistic		0.171	0.092
Asymp. Sig. (2-tailed)		0.018	0.200

CONCLUSION

This paper examines the relationship between average per capita household monetary income and average per capita consumption of bakery products, eggs and potatoes in Bulgaria for the quarters 2016-2023, respectively. The following conclusions are drawn:

1) The correlation between income and potato consumption is low and statistically insignificant, therefore it has not been investigated further.

2) There is a strong and significant negative correlation between income and consumption of bakery products on average per capita of the household, indicating that as income increases, consumption of bakery products decreases. This is to be expected given that people with higher incomes can afford to buy more expensive food products and prefer them to bakery products. The constructed inverse model for bakery products consumption shows that the minimum level

of pasta consumption is 14.729 kg per person per household per quarter, or about 4.91 kg per person per month. This amounts to 163.67 g per day, which in turn equates to about 5-6 slices of bread (1 slice of bread varies between 25 and 35 g) per day.

3) There is a moderate, significant and positive correlation between income and egg consumption on average per household person. This is an indicator that as income increases, egg consumption also increases. Accordingly, the constructed inverse model for egg consumption shows that the income threshold below which eggs are not purchased is BGN 285.985 per person in the household per quarter or BGN 95.33 per month. The upper level of egg consumption that is not exceeded - regardless of income - is 45.852 eggs per person per household per quarter. This equates to 15 eggs per person per month. As far as health standards are concerned, it is estimated that the average healthy person can consume up to 7 eggs per week [6], which equates to around 28 eggs per month.

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REFERENCES

- [1] Montclair State University, <<https://msuweb.montclair.edu/~lebel/PSC643IntPolEcon/EngelCurves.pdf>>.
- [2] Greelane, <greelane.com/>.
- [3] Chai, A., & Moneta, A., 2010. Retrospectives: engel curves. *Journal of Economic Perspectives*, 24(1), 225-240, <<https://pubs.aeaweb.org/doi/pdf/10.1257/jep.24.1.225>>.
- [4] Gujarati, D., 2005. *Basic Econometrics*. Fourth edition, Mc – Graw – Hill, New York.
- [5] National Statistical Institute, < <https://www.nsi.bg/>>.
- [6] Harvard Health Publishing, < https://www.health.harvard.edu/newsletter_article/how-many-eggs-can-i-safely-eat>.