FRI- 20.21-1-TMS-04

METHODOLOGY FOR DETERMINING PERMISSIBLE CONTACT STRESSES AND GEAR MODULE IN A SINGLE-STAGE 3:1 SPUR GEAR REDUCER [4]

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Abstract: *This work presents a design and verification workflow for a single-stage spur gear reducer that centers on determining permissible contact stresses and selecting the gear module, then carrying the result into a parametric CAD environment. The approach begins with an analytical pre-sizing based on Hertzian contact theory and the methodology used in standard gear-design practice. Material selection favors case-hardened alloy steels with ground tooth flanks; allowable stresses are obtained from tabulated endurance data and reduced by safety and technological factors. The pre-sized geometry (module, tooth counts, face width, center distance) is then mapped to a fully parametric SolidWorks model using equations and design tables so that all key dimensions remain associative.*

Keywords: *Spur gear reducer, Load distribution, Contact stress, CAD*

INTRODUCTION

Designing involute cylindrical gears remains a cornerstone task in machine elements, yet the way contact strength is calculated varies across standards and local practice. Comparative studies in the literature highlight notable methodological differences between GOST, BDS, and ISO approaches to surface durability, which can lead to divergent design decisions for the same duty and materials (Angelova, Varbanov, & Ronkova, 2012; Mollova & Dobreva, 2018). At the same time, foundational Bulgarian textbooks provide a consistent classroom pathway—material selection, sizing rules, and verification checks—that practitioners still rely on (Nenov, Andreev, Stamatov, & Spasov, 2007; Stoyanov, Lesev, & Stoykov, n.d.). This mixed landscape underscores the need for a unified workflow that preserves local educational conventions while aligning rigorously with ISO 6336.

This paper proposes a contact-stress–driven sizing and verification workflow for a single-stage involute spur gear reducer that harmonizes tabulated endurance concepts from ISO 6336 formulation for surface durability. The approach begins with an analytical pre-dimensioning step grounded in Hertzian logic and proceeds to a formal ISO check that explicitly accounts for mesh overlap and load distribution factors. Crucially, the workflow is implemented as a parametric SolidWorks model, so analytical assumptions, discrete standard choices (module series, tooth counts, and rounded center distance), and verification results remain traceable and regenerable inside the CAD environment.

Methodologically, the paper formalizes two bridges that are often treated informally. First, it links a rapid cubic sizing criterion for the minimum pinion diameter to the ISO contact-stress expression by consistent factor grouping, ensuring that the "fast" estimate and the "full" check are algebraically compatible. Second, it codifies the discretization from continuous results to standard modules and integer tooth counts—together with center-distance rounding and profile-shift management—in a way that mirrors the didactic sequencing found in Bulgarian course design (Nenov et al., 2007; Stoyanov et al., n.d.). Service-life considerations and reliability arguments are framed in terms of practical applications, drawing on

[4] Докладът е представен на 24 октомври 2025 г. в научната секция „Транспорт и машинознание“ с оригинално заглавие на български език: Методология За Определяне На Допустими Контактни Напрежения И Модул На Зъбната Предавка В Едностъпен 3:1 Редуктор С Целостно Зъбна Предавка

insights from traction gearbox studies to motivate the choice of duty and safety factors (Avdzhieva & Staevski, 2005).

The contributions are threefold: (I) an integrated, ISO-consistent pipeline from quick sizing to formal contact verification; (II) a parametric CAD realization that ties analytical variables to SolidWorks equations and design tables; and (III) a structured sensitivity view on overlap and load-distribution factors to explain safety margins under realistic variation. The remainder of the paper reviews pertinent standards and local methodologies, details the proposed procedure and its CAD mapping, presents a worked reducer design with verification outcomes, and discusses implications for education and industrial practice, including avenues for bending-fatigue checks, lubrication/thermal sizing, and helical-gear extensions.

The aim of the study is to identify research opportunities in the field of evolutionary entanglement:

Assignment

- **Output Power:** $P_2 = 4{,}1$ kW
- Small/medium industrial class – typical for reducers on conveyors, agitators, packaging machines, etc.;
- **RPM:** $n_1 = 900\ min^{-1}$, $n_2 = 300\ min^{-1}$

Gear ratio: u=3 3:1 reduction is widespread;

- **Life:** C=10 000 h working period;

Typical for a training project and real small gearboxes (several years of intermittent operation).

Purpose – they serve to transmit and convert energy from the power machine (engine) to the working machine.

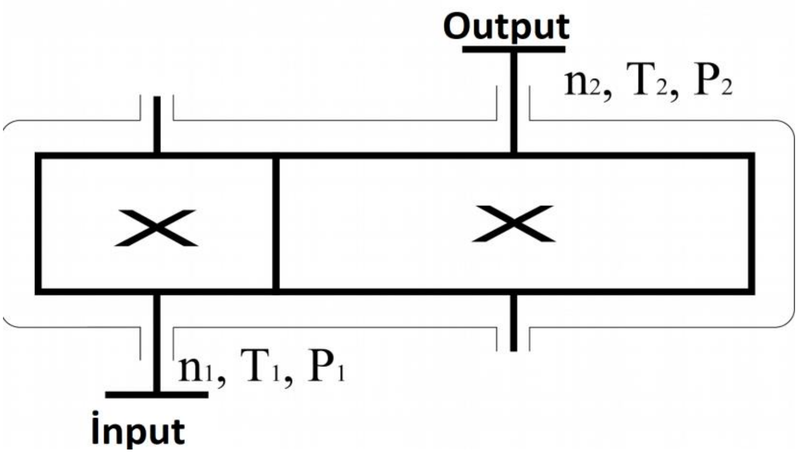

Fig.1. Diagram of the Spur Gear

Tab. 1. Data for the specified (single-stage reducer)

P_2, (kW)	n_1, min^{-1}, (rpm)	n_2, min^{-1}, (rpm)	Life, (h)
4.1	900	300	10000

METHODOLOGY

Below is represented contact-strength sizing and verification of a single-stage spur gear pair.

Analytical workflow (ISO-consistent) with final adopted geometry and loads.

1. Allowable Stresses

Task: Single-stage spur gear transmission with ratio u = 3; hardened and ground teeth (18ХГТ, HRC 60) on both gears.

Base surface endurance (tabulated): $\sigma_{\{H\lim b\}} = 23 \cdot HRC.$

Minimum required safety: $S_{\{H\ min\}} = 1.3$ **lowable contact stress used in verification:**

$$\sigma_{HP} = \frac{\sigma_{H,limb} Z_H Z_L Z_R Z_V Z_W Z_X}{S_{H,min}} \quad (1)$$

For hardened, ground gears and the adopted life, $Z_H \approx 1$ *and the technological product is taken as 1.*

Minimum permissible safety factor of coatings: pitting strength $S_{H_{\min}}$

From the standards for dimensioning gears:

ISO 6336 (the international standard) and AGMA (the American standard), the minimum factor $S_{H_{\min}}$ is agreed between the manufacturer and the customer, and the note refers to ISO 6336-1, p. 4.1.7 for comments on the choice of safety.

The ISO standard defines $S_{H_{\min}}$ and requires $S_H > S_{H_{\min}}$, but does not set a universal value.

The German standard DIN 3990-21 (the German standard for "high-speed" gears) offers a specific numerical recommendation.

In section 4.11 "Minimum pothole safety" for the scope of this standard it is written: $S_{H_{\min}} = 1.3$

2. Fast Sizing: Minimum Pitch Diameters

For spur gears (β = 0°) with relative face width $\psi_{\{bd1\}} = \frac{b_w}{d_1}$ and aggregated load factor K_H:

$$d_{1,min} \geq f_H \sqrt[3]{\frac{T_{in}(u+1)K_H}{\psi_{\{bd1\}}\ \sigma_{HP}^2 u}} \quad (2)$$

3. Final Adopted Geometry and Kinematics

Adopted: $m = 2\ mm, z1 = 20, z2 = 60, a_w = a = 80\ mm, \beta = 0°, \alpha = 20°$, ground flanks.

$$d_1 = mz_1, \qquad d_2 = mz_2 \quad (3)$$

$$a = \frac{(z_1+z_2)m}{2cos\beta} => a_w = a(spur) \quad (4)$$

$$d_{a(1,2)} = d_{1,2} + 2m, \qquad d_{f(1,2)} = d_{1,2} - 2(h_a^* + c^*)m \quad (5)$$

$$d_{b(1,2)} = d_{1,2} cos\alpha \quad (6)$$

Numerical values with the adopted data: $d1 = 40\ mm;\ d2 = 120\ mm;\ da1 = 44\ mm;\ da2 = 124\ mm;\ df1 = 35\ mm;\ df2 = 115\ mm;\ db1 = 37.59\ mm;\ db2 = 112.77\ mm.$

4. Covering (Overlap) and Angles

$$\alpha_{a1} = \arccos\left(\frac{d_{b1}}{d_{a1}}\right), \quad \alpha_{a2} = \arccos\left(\frac{d_{b2}}{d_{a2}}\right), \alpha_{tw} = 20° \quad (7)$$

$$\varepsilon_\alpha = \frac{z_1(tan\alpha_{a1} - tan\alpha_{tw}) + z_2(tan\alpha_{a2} - tan\alpha_{tw})}{2\pi}, \qquad \varepsilon_\beta = 0\ (spur) \quad (8)$$

Evaluated overlap: $\varepsilon_\alpha \approx 1.67;\ \varepsilon_\beta = 0.$

5. Forces at the Pitch Circle

$$F_t = \frac{2T_{in}}{d_{w1}}, \quad F_r = F_t tan\alpha_{tw}, \quad F_a = F_t tan\beta, \quad F_n = \frac{F_t}{cos\alpha_{tw} cos\beta} \quad (9)$$

With $T_{in} \approx 44.9\ N \cdot m\ and\ d_{\{w1\}} \approx d1: Ft \approx 2245\ N;\ Fr \approx 817\ N;\ Fa = 0\ N\ (\beta = 0°);\ Fn \approx 2389\ N.$

6. ISO-Consistent Contact Stress Check

Geometric factors: $Z\beta = 1\ (spur), ZH \approx 2.5, ZE \approx 190\ (MPa)^{\{\frac{1}{2}\}}, Z\varepsilon \approx 0.89\ (from\ \varepsilon\alpha).$

$Load\ factors: KA = 1.20, KHv = 1.10, KH\beta = 1.065, KH\alpha = 1.00.$

$$\sigma_H = Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t(u_d+1)}{b_w d_1 u_d} K_A K_{Hv} K_{H\beta} K_{H\alpha}}$$

Computed result with the adopted geometry and factors: $\sigma H \approx 723\ MPa;\ allowable\ \sigma HP \approx 1061.5\ MPa \rightarrow SH \approx 1.47\ (> 1.3).$

7. Summary Table for CAD/CAE (SolidWorks Equations)

Tabl. 2. CAD/CAE entry data (SOLIDWORKS Equations)

Parameter	Symbol	Value
Module	m	2 mm
Teeth (pinion/gear)	z1 / z2	20 / 60
Center distance	a_w	80 mm
Face width	b_w	36 mm
Pitch diameters	d1 / d2	40 / 120 mm
Addendum diameters	da1 / da2	44 / 124 mm
Dedendum diameters	df1 / df2	35 / 115 mm
Base diameters	db1 / db2	37.59 / 112.77 mm
Tangential force	Ft	2245 N
Radial force	Fr	817 N
Normal force	Fn	2389 N
Factors	Zε; Zβ; ZH; ZE	0.89; 1.00; 2.5; 190
Load factors	KA; KHv; KHβ; KHα	1.20; 1.10; 1.065; 1.00
Allowable contact	σHP	1061.54 MPa
Computed contact	σH	723 MPa
Safety	SH	1.47

CONCLUSION

This study demonstrated a coherent, ISO-consistent workflow for sizing and verifying an involute spur gear pair by coupling a rapid contact-strength pre-dimensioning step with a formal ISO 6336 check and a parametric SolidWorks implementation. Using case-hardened, ground steel gears and standard discrete choices (module, tooth counts, rounded center distance), the final geometry satisfied the surface-durability criterion with a comfortable safety margin; all key dimensions and verification factors were bound to CAD equations, ensuring traceable updates from assumptions to drawings. The approach's main contribution is procedural: it bridges the ($\boldsymbol{f_H}$)-based cubic estimator for the minimum pinion diameter to the full Hertzian/ISO formulation, and codifies the discretization rules engineers actually use in production (standard modules, integer teeth, and center-distance rounding), while exposing sensitivity to overlap and load-distribution factors. The present work is limited to contact fatigue; bending fatigue, thermal/lubrication sizing, tolerance-driven accuracy (ISO 1328), NVH, and dynamic load refinement remain to be addressed. Future work will extend the pipeline to ISO 6336-3 bending verification, helical variants with axial loads, duty-cycle reliability, and CAE validation, preserving the same analytical-to-CAD "digital thread" to accelerate design iteration and improve manufacturability.

REFERENCES

Rothemund, M., Otto, M., & Stahl, K. "Applicability of Classic Analytical Calculation Approaches for the Design of Plastic Gears." Forschung im Ingenieurwesen, 87, 987–995, 2023.

Vijayarangan, S., & Ganesan, N. "Static Contact Stress Analysis of a Spur Gear Tooth Using the Finite Element Method, Including Frictional Effects." Computers & Structures, 51(6), 765–770, 1994.

Avdzhieva, T., & Staevski, K. Assessment of the Minimum Service Life of Gearwheels from the Traction Gearbox of a Mainline Locomotive. Mechanics. Transport. Communications, no. 1, 2005, article No. 0049. (***Оригинално заглавие:*** *Авджиева, Т.; Стаевски, К. Оценяване на минималния ресурс на зъбни колела от тягов редуктор от магистрален локомоtив. Механика. Транспорт. Комуникации, бр. 1, 2005, статия №0049.*)

Angelova, E., Varbanov, V., & Ronkova, V. Strength Calculation of Involute Cylindrical Gear Transmissions: Comparison of the GOST, BDS and ISO Standards. Proceedings of the University of Ruse, vol. 51, series 4, 2012, pp. 192–197. (***Оригинално заглавие:*** *Ангелова, Е.; Върбанов, В.; Ронкова, В. „Якостно изчисляване на еволвентни цилиндрични зъбни предавки, съпоставяне на стандартите ГОСТ, БДС и ISO. Научни трудове на Русенския университет, т. 51, серия 4, 2012, с. 192–197.*)

Mollova, G., & Dobreva, A. Differences in the Calculation of Involute Cylindrical Gears in Three Main Standards (BDS 17108-89, ISO 6336, GOST 21354-87). Proceedings of the University of Ruse, 2018, vol. 57, book 4, pp. 25–29. (***Оригинално заглавие:*** *Моллова, Г.; Добрева, А. „Различия в изчисляването на еволвентни цилиндрични предавки в три основни стандарта (BDS 17108-89, ISO 6336, GOST 21354-87). Proceedings of University of Ruse, 2018, vol. 57, book 4, с. 25–29.*)

Nenov, P., Andreev, D., Stamatov, P., & Spasov, I. Course Design in Machine Elements. Sofia: Tekhnika, 2007. (***Оригинално заглавие:*** *Ненов, П.; Андреев, Д.; Стаматов, П.; Спасов, И. Курсово проектиране по машинни елементи. София: Техника, 2007.*)

Stoyanov, S., Lesev, I., & Stoykov, D. Machine Elements. Sofia: Tekhnika. (***Оригинално заглавие:*** *Стоянов, С.; Лесев, И.; Стойков, Д. Машинни елементи. София: Техника.*)

ISO 6336-1, *Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors.*

ISO 6336-2, *Calculation of load capacity of spur and helical gears — Part 2: Calculation of surface durability (pitting).*

ISO 6336-3, *Calculation of load capacity of spur and helical gears — Part 3: Calculation of tooth bending strength.*

ISO 6336-5, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of materials.*

ISO 6336-6, *Calculation of load capacity of spur and helical gears — Part 6: Calculation of service life under variable loading.*

ISO 1328-1, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth.*

ISO 21771, *Gears — Cylindrical involute gears and gear pairs — Concepts and geometry.*