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REED-SOLOMON CODES IN EDUCATION: FROM THEORY TO PRACTICAL APPLICATIONS IN DIGITAL TECHNOLOGIES [1]

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Abstract: *This paper examines Reed-Solomon codes as a key tool for error correction in digital technologies and their place in the educational process. The basic mathematical principles of codes based on polynomials over finite fields are presented, as well as examples of their wide application – from compact discs and QR codes to space communications. Special attention is paid to the connection with education – how the study of these codes supports the understanding of algebra, computer science, and information technology, as well as what educational examples and exercises can be used in the classroom. The advantages for the educational process are emphasized – motivating students through real-world applications, building cross-curricular connections, and developing analytical thinking. The paper shows that Reed-Solomon codes are not only a mathematical achievement but also a valuable resource for modern education.*

Keywords: *Reed-Solomon codes, Error correction, Data redundancy, Polynomials, Interpolation, Digital communication, Computer science, Education, Learning process, CD, DVD, Media, QR codes, Digital video broadcasting (DVB), Information reliability, Mathematical modelling, Practical applications.*

INTRODUCTION

In the modern world, information is one of the most valuable resources. Huge amounts of data are transferred daily via the Internet, satellite connections, and digital media. One of the main problems in the transmission and storage of information is the possibility of errors (for example, due to noise in the channel or physical defects in the medium).

To ensure reliability, error correction codes are used. Among them, the Reed-Solomon codes, developed in 1960 by Irwin Reed and Gustav Solomon (Reed, I. S., & Solomon, G., 1960), occupy a special place. Their strength lies in the ability to detect and correct groups of errors, which makes them extremely effective in real conditions (Wicker, S., & Bhargava, V., 2009), (Wicker, S., & Bhargava, V., 1999).

The topic is relevant for the educational process because it combines mathematical theory and practical applications that students encounter daily.

THEORETICAL PART

Reed-Solomon codes are a type of block error-correcting code based on polynomials over finite fields. The basic idea is to represent the message as a polynomial that is evaluated over

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certain values. If some of the values are corrupted, it is possible to recover the polynomial, and with it the original message. For example, if we have 4 symbols of information, we can add 2 more check symbols, so that if up to 2 symbols are corrupted, they can be repaired. This is a powerful tool because it uses a strictly mathematical basis, but the results are directly applicable in practice (A Little Bit about Reed-Solomon Codes, 2025), (Geisel, W.A., 1990).

APPLICATION OF CODES

Some of the applications of Reed-Solomon codes are presented below in the paper (Reed–Solomon error correction, 2025).

Digital media

CDs, DVDs, and Blu-ray discs use these codes to correct scratches and defects (Fig. 1 a).

QR codes

Thanks to Reed-Solomon codes, we can scan damaged or partially obscured QR codes (Fig. 1 b) (QR code, 2025).

Telecommunications in space

NASA uses Reed-Solomon codes to transmit data from probes and satellites (Fig. 1 c) (Hagenauer, J., Offer, E., & Papke, L., 1993).

Mobile communications and internet protocols

Reed-Solomon codes ensure reliability in noisy channels (Fig. 1 d) (Sachs, J., 2018).

Digital Video Broadcasting (DVB)

Reed-Solomon codes are a fundamental element in digital television and radio broadcasting standards. They protect the video signal from loss of quality when transmitted over noisy channels and provide a stable picture and sound even in the presence of interference or a weak antenna. Thanks to them, digital television reaches millions of users with high quality and reliability (Fig. 1 e).

These examples are easily recognizable to students and help to understand the practical value of mathematical concepts.

a) b) c) d) e)

Fig. 1. Some applications of Reed-Solomon codes: a) CDs; b) QR codes; c) space communications; d) mobile communications; e) digital video broadcasting

CONNECTION WITH THE LEARNING PROCESS

The study of Reed-Solomon codes is a valuable tool for modern education in several areas.

Math

The topic is suitable for studying polynomials, division with remainders, systems of equations, and finite field theory. Students see that these "abstract" topics have concrete practical value (Fig. 2 a).

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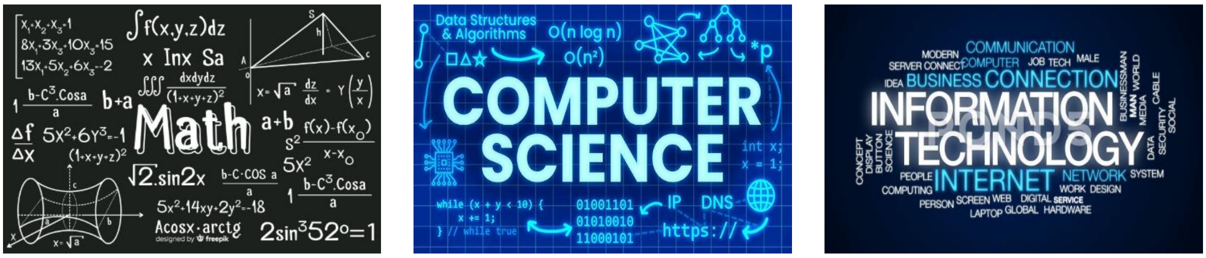
Computer Sciences

It demonstrates how algorithms and data are closely related to mathematical principles. Teachers can use examples with real code for encoding and decoding (Fig. 2 b).

Information Technologies

It shows students that the reliability of digital devices and networks is achieved through mathematical methods. This develops an understanding of the structure of modern communication systems (Fig. 2 c).

Thus, Reed-Solomon codes become an example of an interdisciplinary connection that unites mathematics, computer science, and information technology.



a) b) c)

Fig. 2. Connection with the learning process: a) Math; b) Computer sciences; c) Information technologies

EDUCATIONAL EXAMPLES

In order to turn theory into practical knowledge, different approaches can be used in the learning process.

Classroom simulations

The teacher can give a short message (for example, a word "КОД" in Bulgarian), which is "sent" to another student with introduced errors. Then, using a simple scheme with added control symbols, the group restores the original message. This shows how the correction works in practice.

I. We convert the letters into numbers. Let's assume that each letter of the Bulgarian alphabet will be written with its corresponding number: К = 11, О = 15, Д = 5. Then the message will be $M = [11, 15, 5]$.

II. The polynomial $P(x) = 11 + 15x + 5x^2$ is formed.

III. Several points are chosen (for example $x = 1, 2, 3, 4$) and the values of $P(x)$ are calculated to add redundancy (encoding), as follows:

$$P(1) = 11 + 15.1 + 5.1^2 = 31 \quad (1)$$

$$P(2) = 11 + 15.2 + 5.2^2 = 61 \quad (2)$$

$$P(3) = 11 + 15.3 + 5.3^2 = 101 \quad (3)$$

$$P(4) = 11 + 15.4 + 5.4^2 = 151 \quad (4)$$

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The message is encoded as a list of these values. These are the information and control symbols taken together $[31, 61, 101, 151]$.

IV. Let us assume that when the message is received, the value for $x = 3$ is corrupted. From the remaining values $x = 1, 2, 4$ we can find the coefficients of the polynomial:

$$\left| \begin{array}{r} a_0 + a_1 + a_2 = 31 \\ a_0 + 2a_1 + 4a_2 = 61 \\ a_0 + 4a_1 + 16a_2 = 151 \end{array} \right. \qquad (5)$$

Subtracting equation 1 from equation 2 in (5) yields a new equation 4 with two unknowns:

$$a_1 + 3a_2 = 30 \qquad (6)$$

Subtracting equation 2 from equation 3 yields another new equation 5 with two unknowns:

$$a_1 + 6a_2 = 45 \qquad (7)$$

To find the coefficient a_2, it is necessary to subtract equation 4 from equation 5:

$$3a_2 = 15 \rightarrow a_2 = \frac{15}{3} = 5 \qquad (8)$$

We also find the remaining coefficients a_0 and a_1 by substituting a_2 in equations 1 and 4:

$$a_1 = 30 - 3.5 = 15 \qquad (9)$$

$$a_0 = 31 - (15 + 5) = 11 \qquad (10)$$

We find the polynomial $P(x) = 11 + 15x + 5x^2$ and restore the lost value at $x = 3$ by substituting into it.

Scan QR code

Students can generate QR codes with different levels of security, then partially cover them or add “noise”. They will see that even when damaged, the QR code often remains readable. This exercise is easy, attractive and close to their experience.

Fig. 3. Using QR codes during the classes

Through programming

In computer science and programming classes, a basic version of Reed-Solomon coding can be implemented. Even in a simplified version (with the addition of control characters), the exercise demonstrates how mathematical ideas are transformed into working algorithms.

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Example: We want to send a message consisting of three characters $M = [7, 2, 5]$.

I. The message is interpreted as the coefficients of a polynomial:

$$P(x) = 7 + 2x + 5x^2 \quad (11)$$

II. We calculate the values of the polynomial for several randomly selected points (for example $x = 1, 2, 3, 4, 5$), which we add as redundancy during encoding:

$$P(1) = 7 + 2.1 + 5.1^2 = 14 \quad (12)$$

$$P(2) = 7 + 2.2 + 5.2^2 = 31 \quad (13)$$

$$P(3) = 7 + 2.3 + 5.3^2 = 58 \quad (14)$$

$$P(4) = 7 + 2.4 + 5.4^2 = 95 \quad (15)$$

$$P(5) = 7 + 2.5 + 5.5^2 = 142 \quad (16)$$

The message is encoded as a list of these values. These are the information and control symbols taken together $[14, 31, 58, 95, 142]$.

III. Let's assume that when the message is received, the value for $x = 3$ is corrupted. Using program code (Fig. 4), the value can be restored:

```
#include <iostream>
using namespace std;
int main() {
    // Known values from the polynomial P(x) = 7 + 2x + 5x^2
    // P(1) = 14, P(2) = 31, P(3) = 58, P(4) = 95, P(5) = 142
    // We assume that the value at x = 3 is corrupted
    int x1 = 1, y1 = 14;
    int x2 = 2, y2 = 31;
    int x3 = 4, y3 = 95;

    // Solving a system of equations for a0 + a1*x + a2*x^2
    // Equations:
    // a0 + a1*1 + a2*1 = 14
    // a0 + a1*2 + a2*4   = 31
    // a0 + a1*4 + a2*16  = 95

    // Step 1: we subtract equations to eliminate a0
    // (equation2 - equation1) => a1 + 3a2 = 17
    // (equation3 - equation2) => 2a1 + 12a2 = 64

    int a2 = 5; // from (2a1 + 12a2 = 64) and (a1 + 3a2 = 17) -> a2=5
    int a1 = 17 - 3*a2;    // => a1 = 2
    int a0 = 14 - a1 - a2; // => a0 = 7

    cout << "Coefficients of the polynomial: " << endl;
    cout << "a0 = " << a0 << ", a1 = " << a1 << ", a2 = " << a2 << endl;

    // Step 2: Recovering the missing value at x=3
    int missing = a0 + a1*3 + a2*3*3;
```

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```
    cout << "Restored value at x = 3: " << missing << endl;

    return 0;
}
```

Fig. 4. Using program code for restoring the value

In this way, the problem is understandable for students because it works with a system of equations and clearly shows how to reconstruct the original polynomial and the missing data from known values.

Project Assignments

Students can explore: “How do you ensure that a music CD plays properly despite scratches?” (Fig. 5) or “Why do QR codes work even when damaged?” Such projects connect learning to their personal experiences.

Fig. 5. “How do you ensure that a music CD plays properly despite scratches?”

ADVANTAGES FOR THE LEARNING PROCESS

Integrating the topic of Reed-Solomon codes into the learning environment brings a number of benefits.

Motivation through real-life examples

Students understand that abstract mathematics is the basis of the technologies they use every day.

Development of analytical thinking

Understanding error correction stimulates logical reasoning and working with algorithms.

Cross-curricular connections

The topic naturally unites mathematics, computer science and information technology, developing a holistic view of science.

Practical training

Students acquire knowledge and skills useful for future professions in the field of digital technologies, engineering and data science.

Creativity and investigative spirit

Through projects and experiments, students are encouraged to seek answers to real-world questions.

CONCLUSION

Reed-Solomon codes uniquely demonstrate the connection between mathematics (including higher mathematics) and information technology in our everyday lives. They make it possible to correct errors in information, without which the modern digital world would not be possible.

Including them in the learning process is not just an introduction to another mathematical technique, but an opportunity to create a bridge between theory and practice. Students not only learn new concepts, but also see how these ideas have a concrete application in the real world. This makes learning more interesting, useful and inspiring for the future.

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